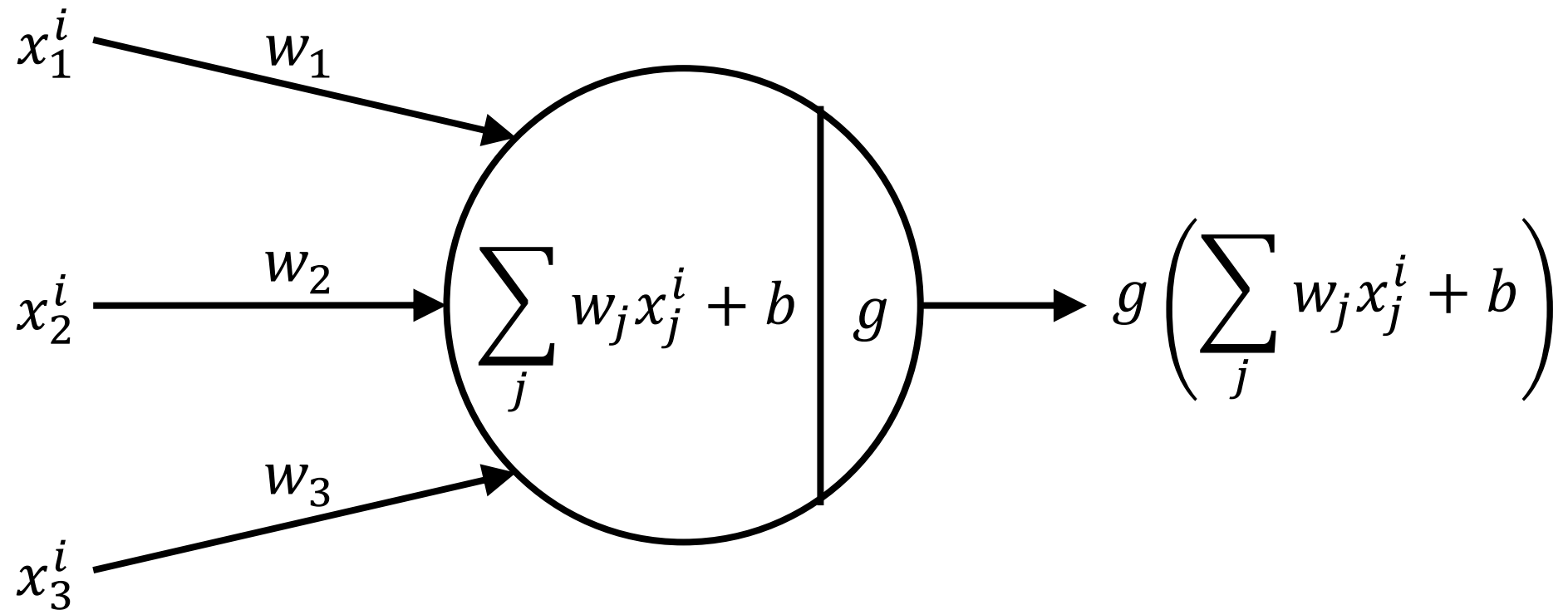


Robot Learning

Basics of computer vision for robotics
Representation learning

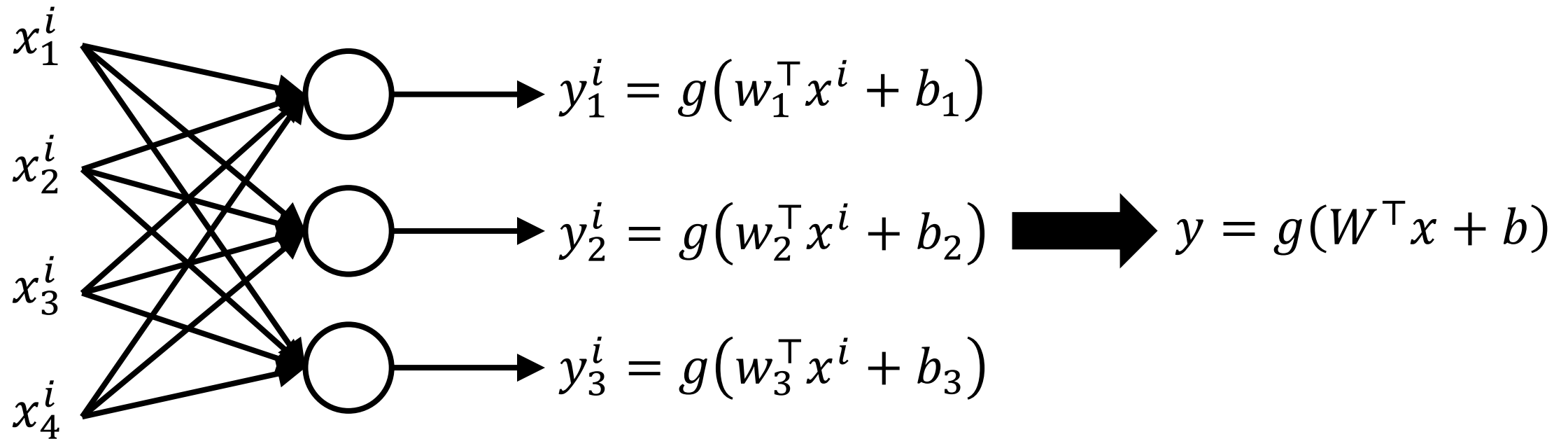
Last time...

1. A perceptron



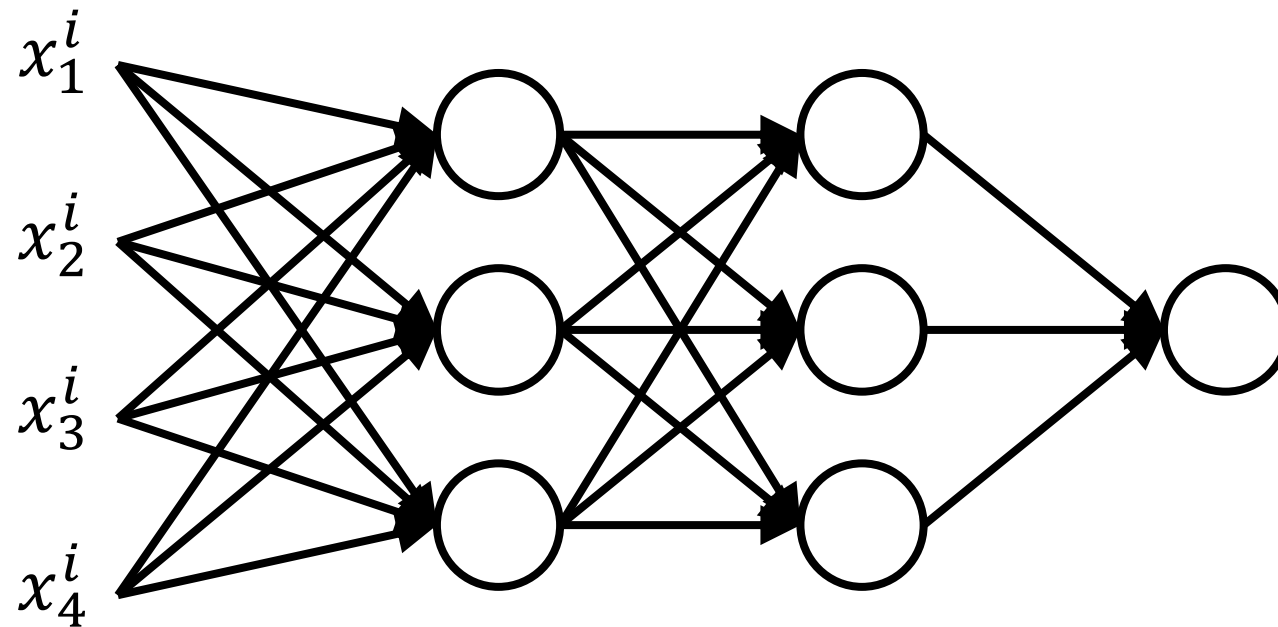
Last time...

2. A single layer neural network

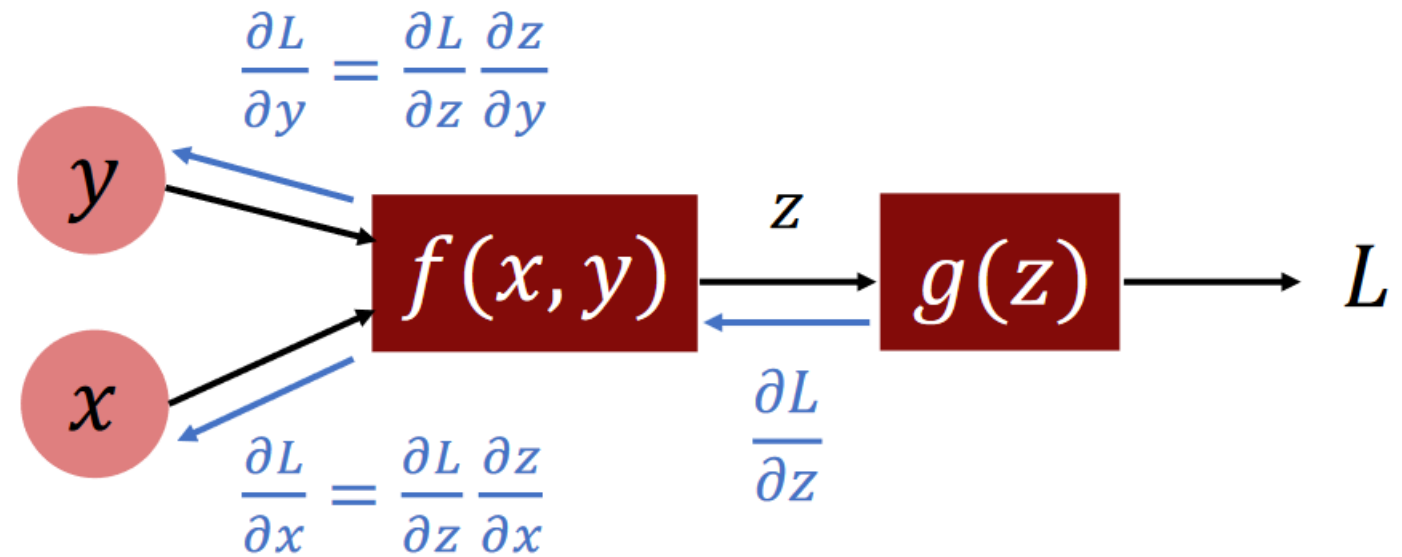


Last time...

3. A deep neural network

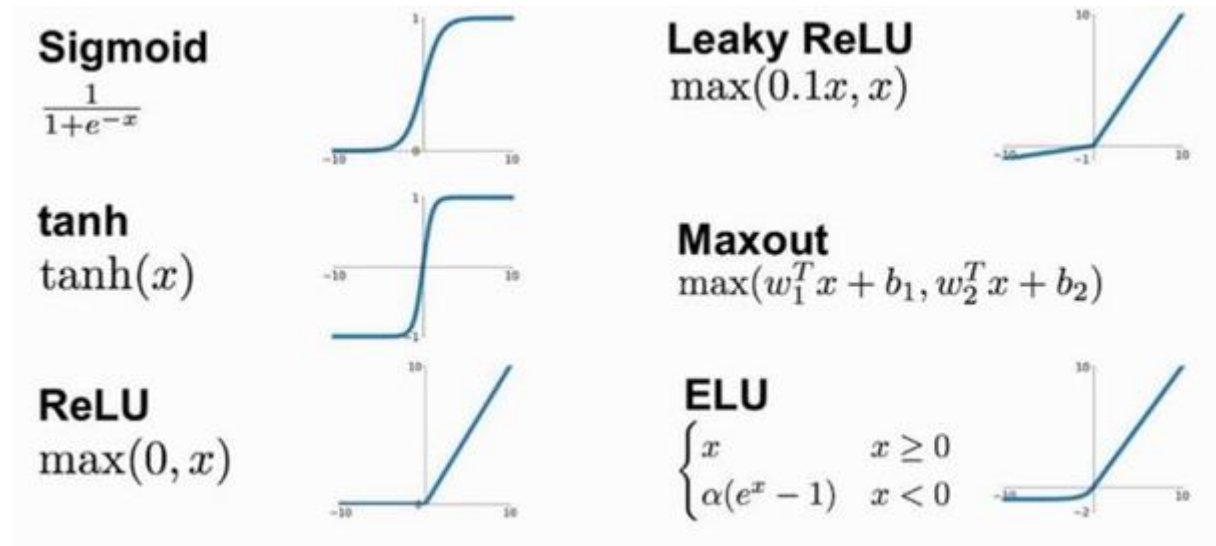


Last time...

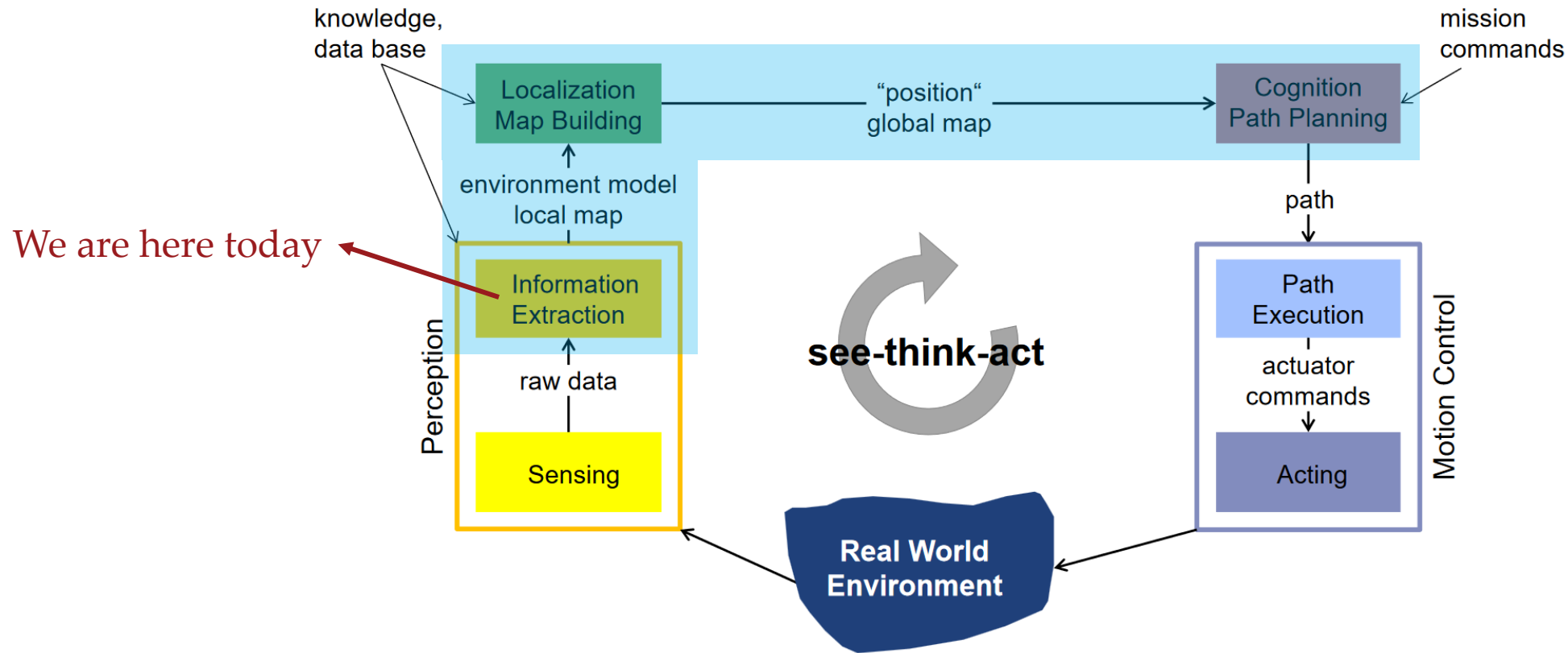


Last time...

g should not be a linear function.

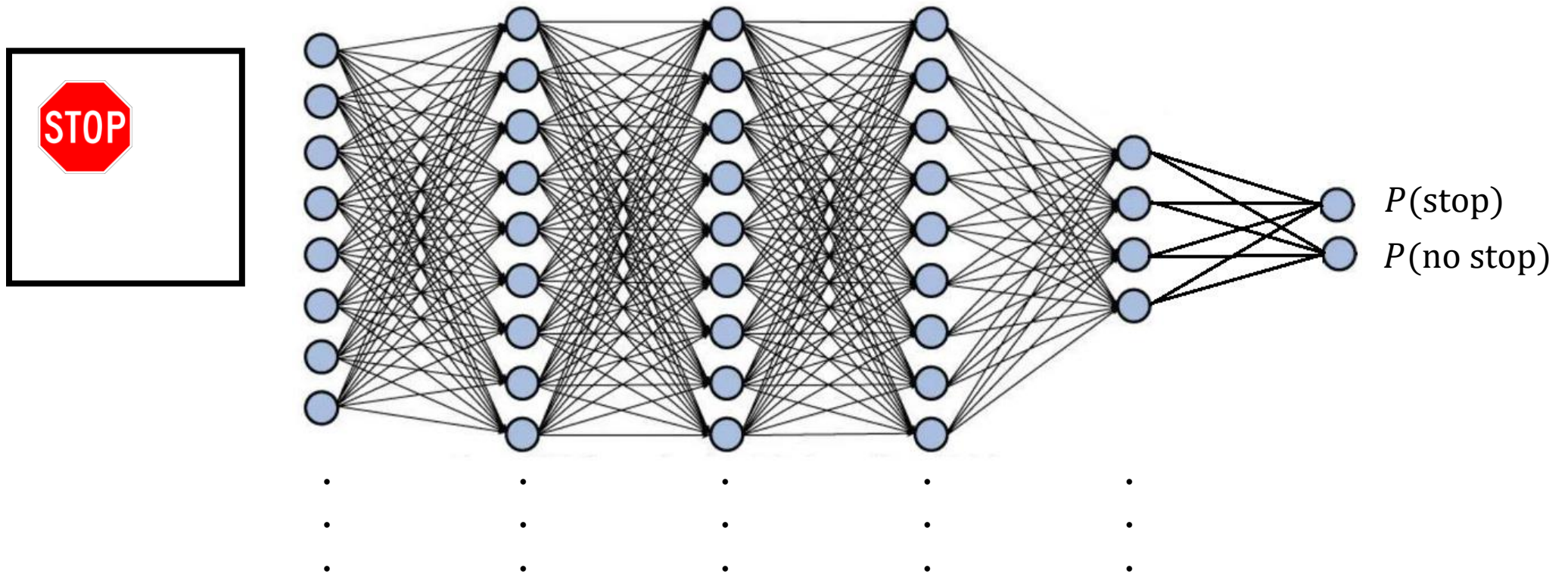


Robot learning



Neural networks for images

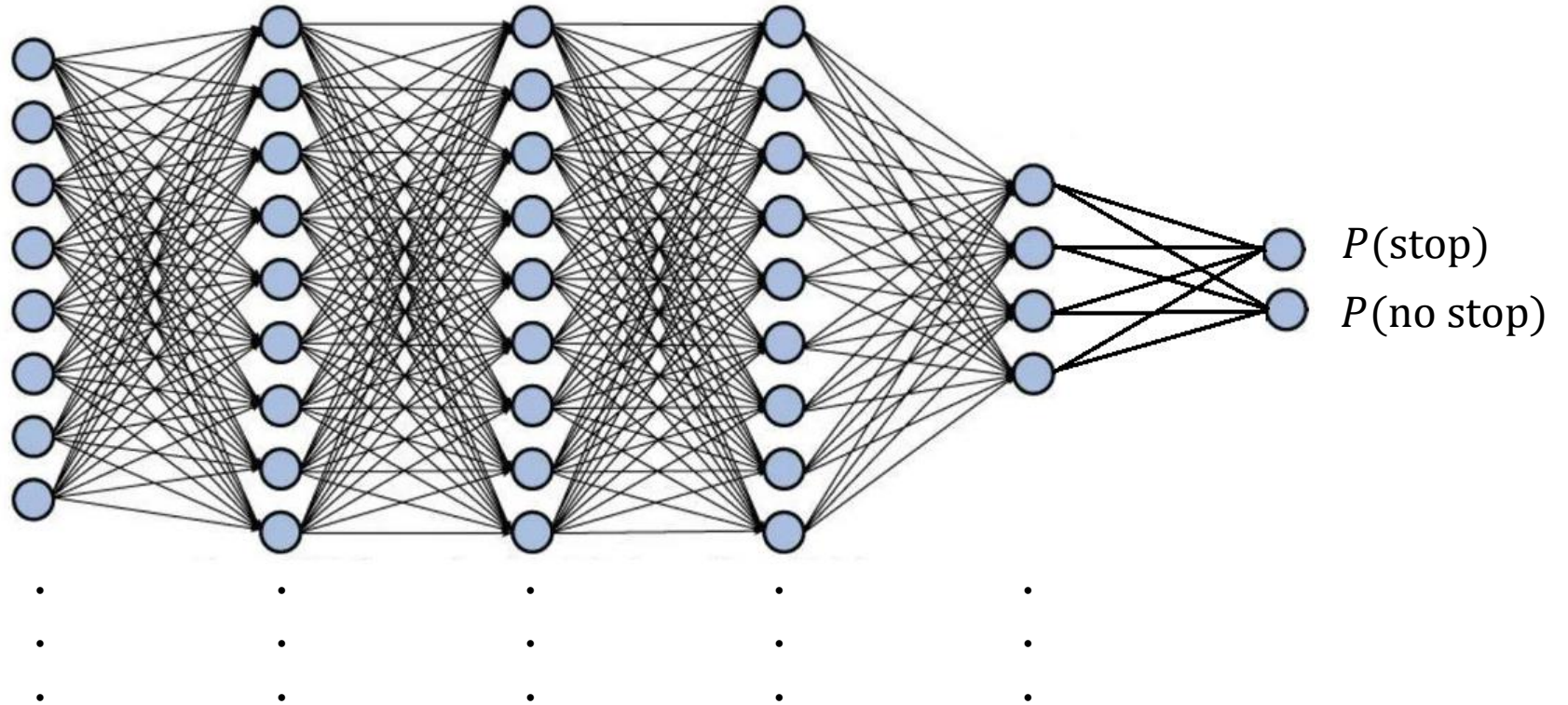
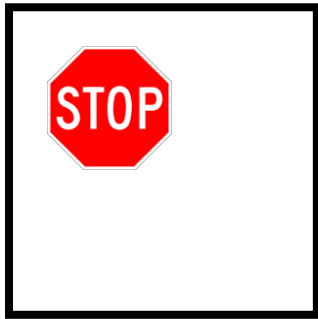
Easy “solution”: flatten the image and make it a vector.



Neural networks for images

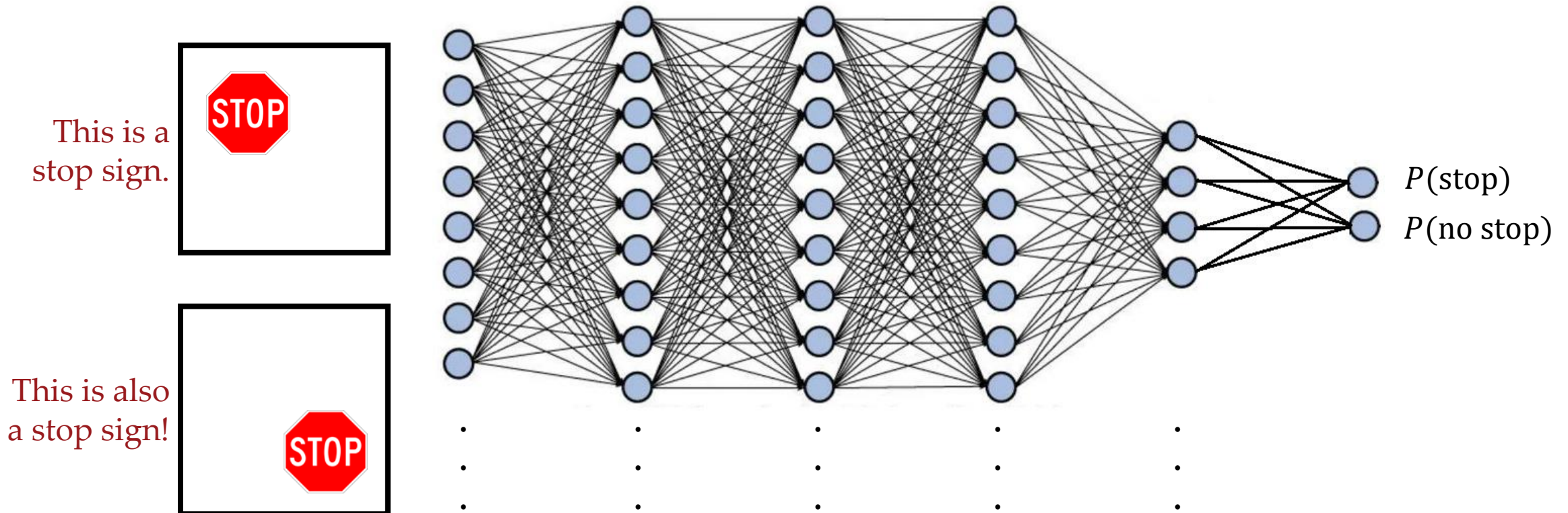
Easy “solution”: flatten the image and make it a vector.

This is a
stop sign.



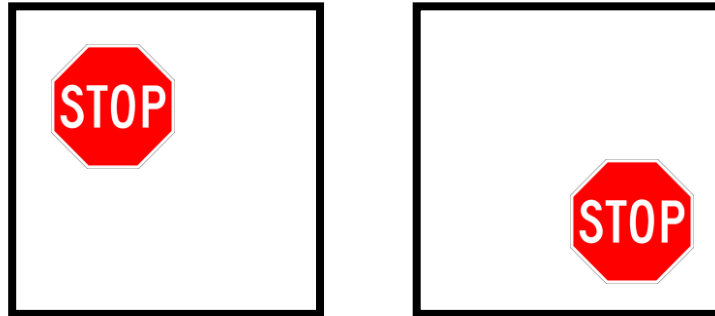
Neural networks for images

Easy “solution”: flatten the image and make it a vector.



Neural networks for images

Regardless of its location in the image,  is a stop sign.



Spatial locality → weight sharing

Convolution

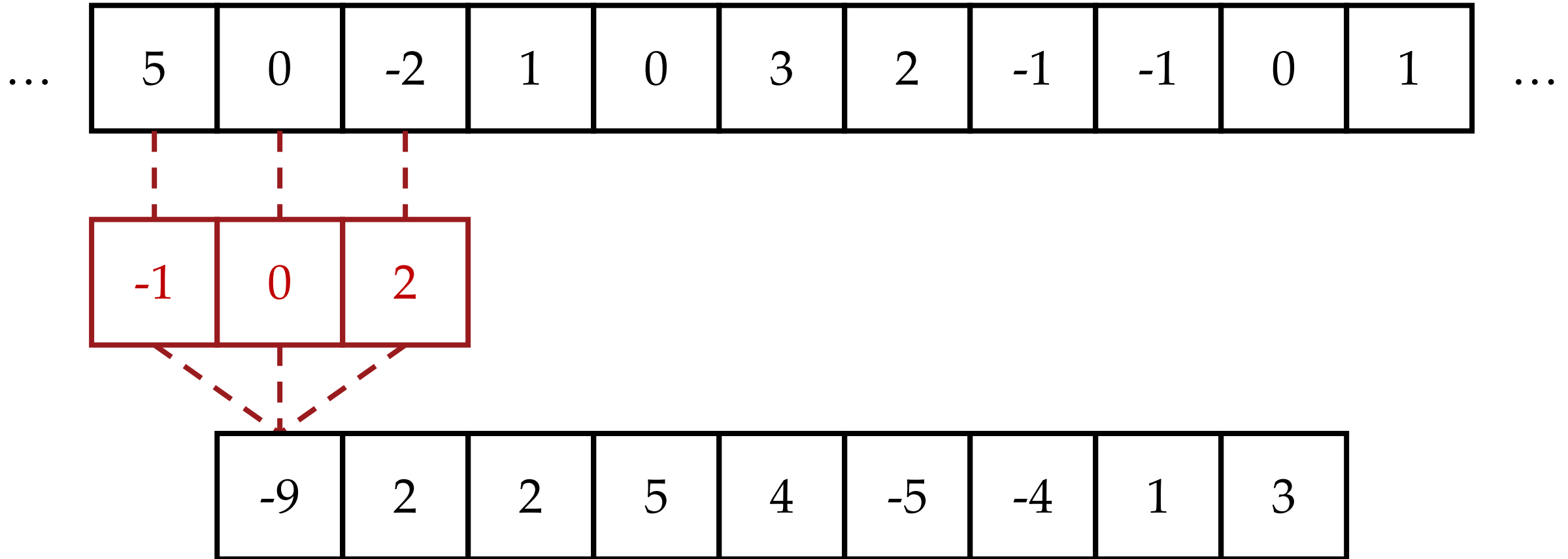
...

5	0	-2	1	0	3	2	-1	-1	0	1
---	---	----	---	---	---	---	----	----	---	---

...

2	0	-1
---	---	----

Convolution



Why is convolution important?

Many major technological developments in the past 50 years somehow relates to convolution.

- Used in: image processing, machine learning, telecommunications, audio processing, medical imaging, radar/sonar systems, seismology, quantum computing, optics, data compression, 3D graphics...

Why is convolution important?

Many major technological developments in the past 50 years somehow relates to convolution.

- Used in: image processing, machine learning, telecommunications, audio processing, medical imaging, radar/sonar systems, seismology, quantum computing, optics, data compression, 3D graphics...

Became especially popular after James Cooley and John Tukey developed Fast Fourier Transform (FFT) in 1965.



Reduces the computational complexity of convolution from $O(NM)$ to $O(N \log N)$

What's more?

FFT is *“the most important numerical algorithm of our lifetime.”*

Gilbert Strang, 1994

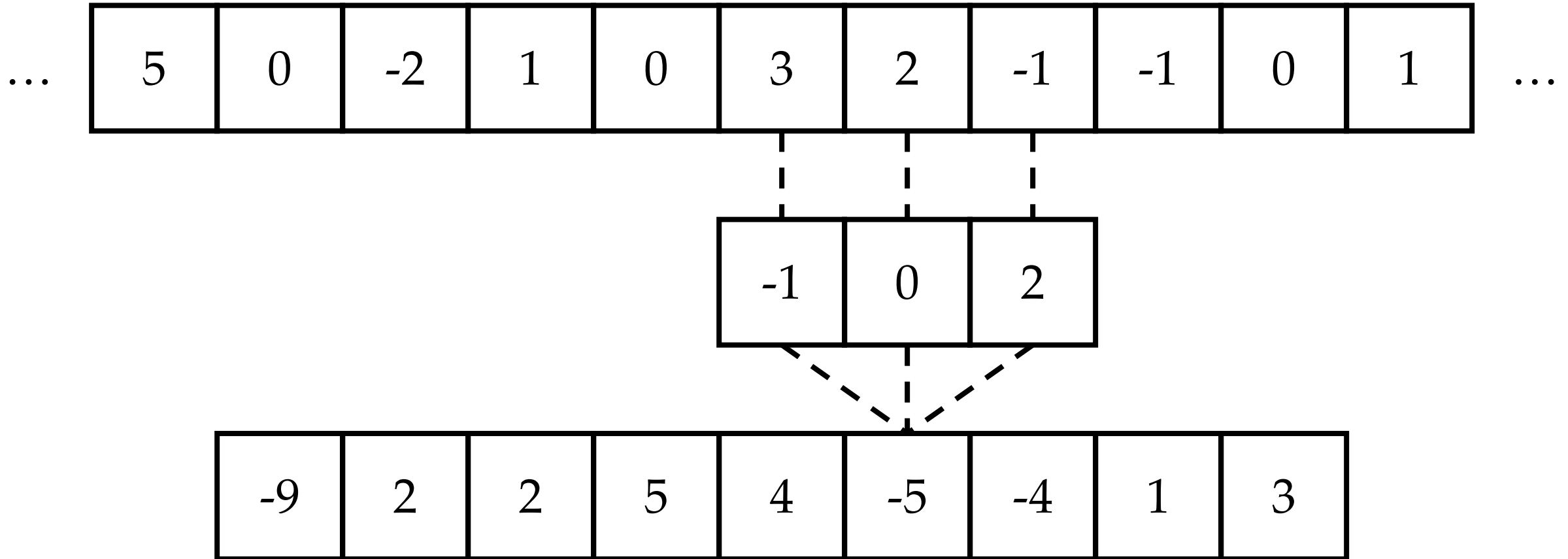
Top 10 algorithms of the 20th century, according to IEEE's Computing in Science & Engineering magazine:

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

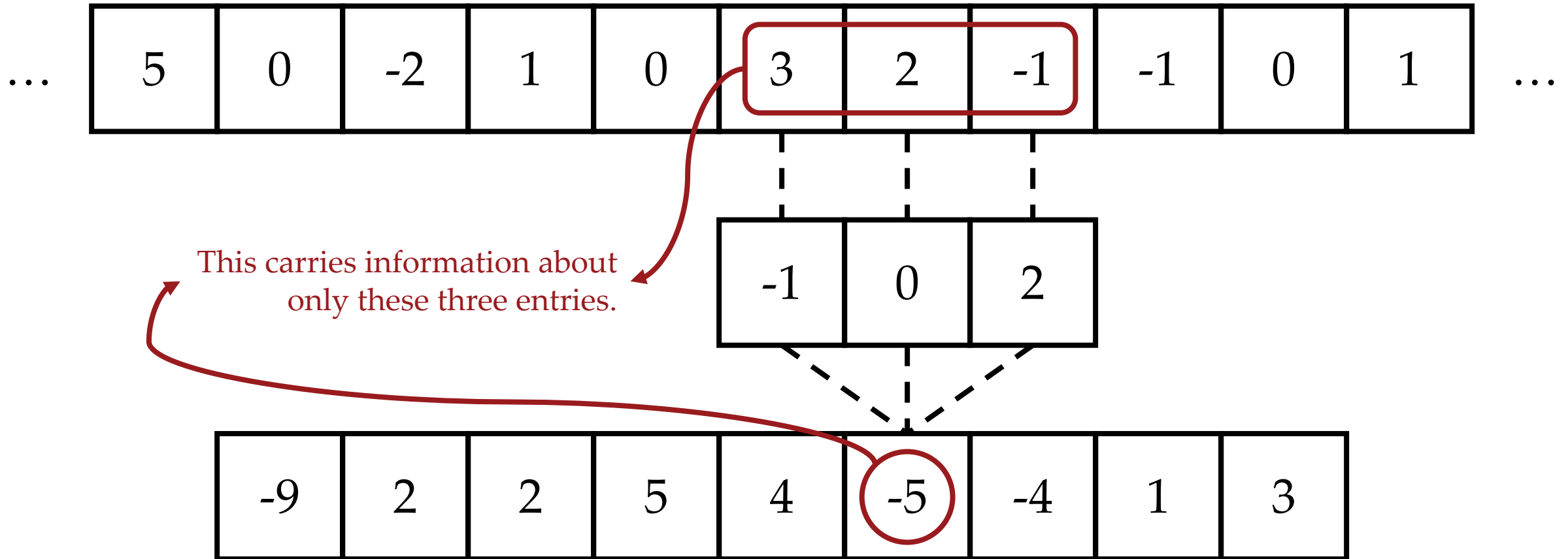
More on convolution and FFT

- EE 301L: Linear Systems
- EE 483: Introduction to Digital Signal Processing
- EE 434Lx: Digital Signal Processing Design Laboratory
- BME 413: Bioengineering Signals and Systems

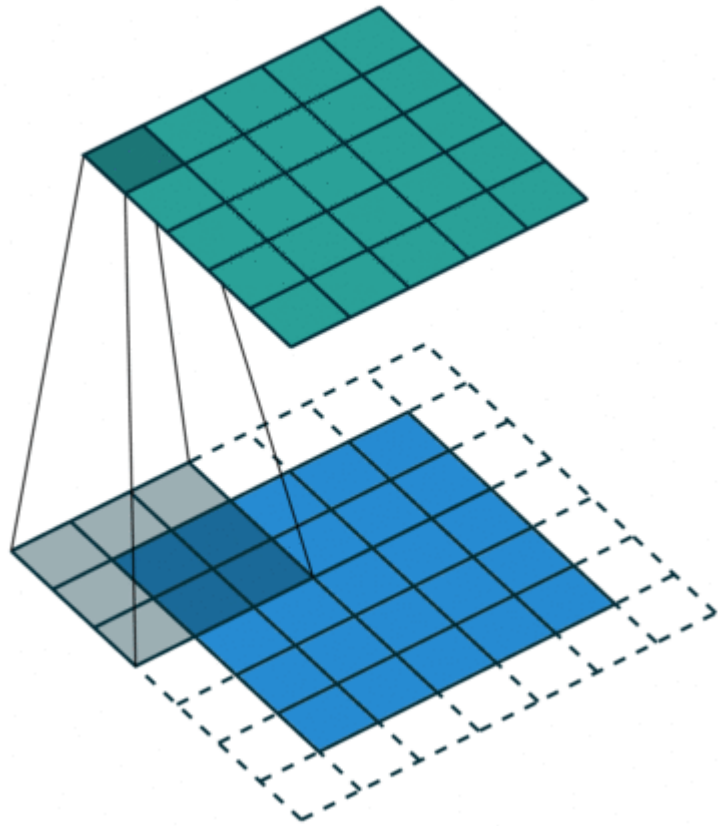
Convolution



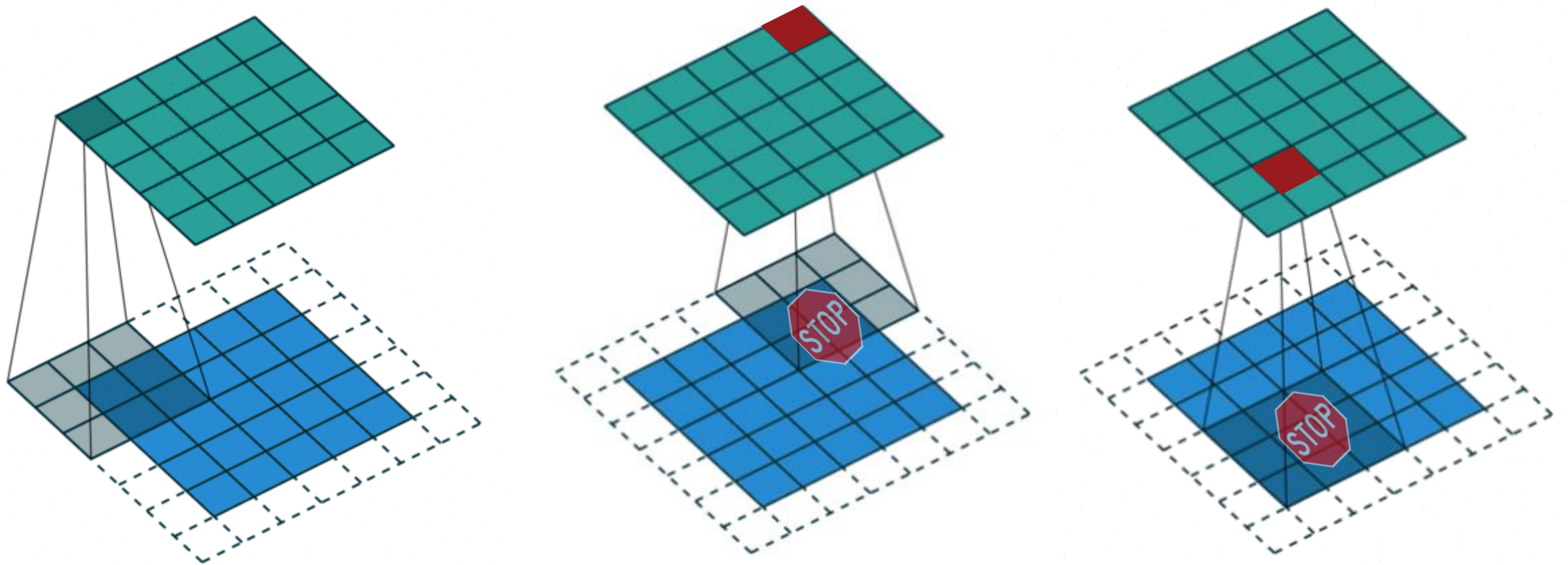
Convolution



Convolution in 2D

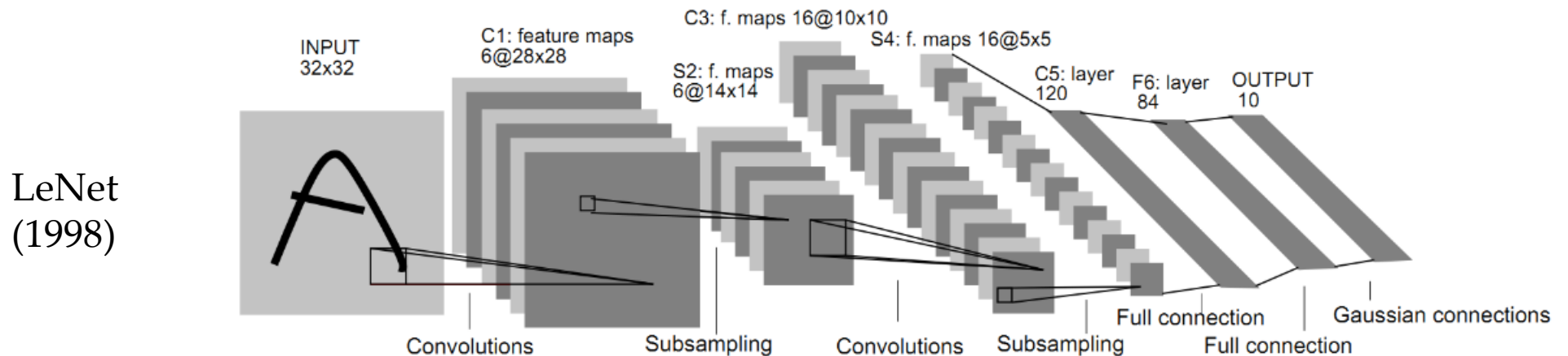
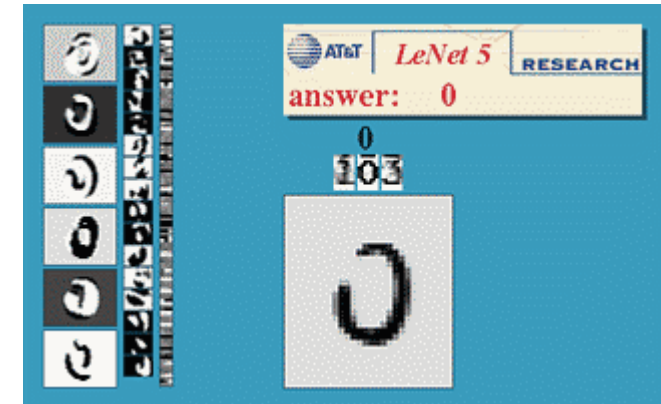


Convolution in 2D

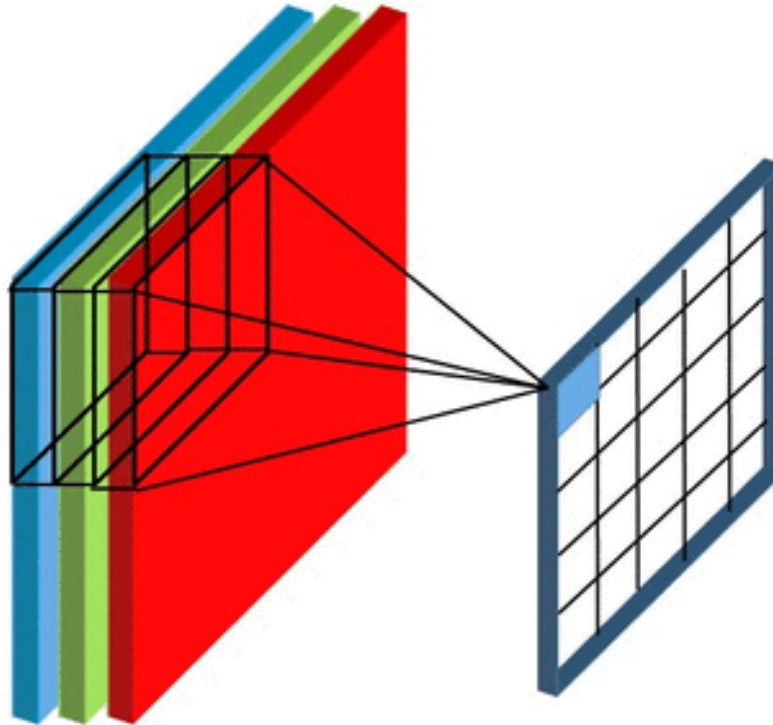


Convolutional neural networks (CNN)

- Traditionally consists of:
 - Convolutional layers
 - Nonlinearity layers
 - Pooling layers
 - Fully-connected layers

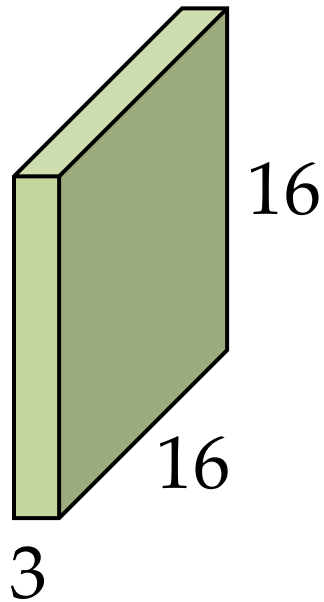


Convolutional layer



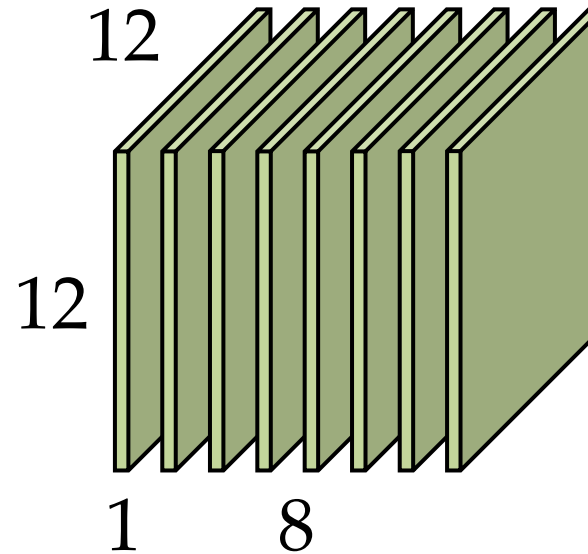
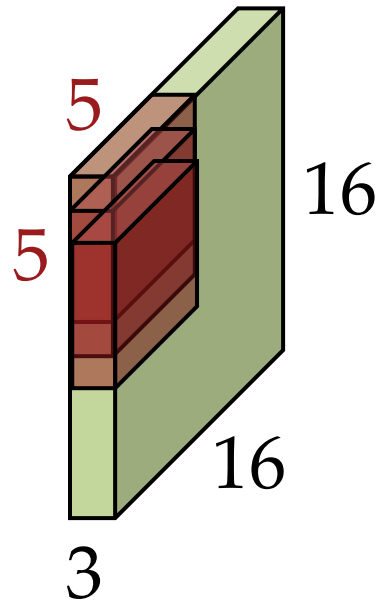
Convolutional layer

Let's say we have an image with size: $(16, 16, 3)$ and 8 kernels where each kernel is $(5, 5)$.

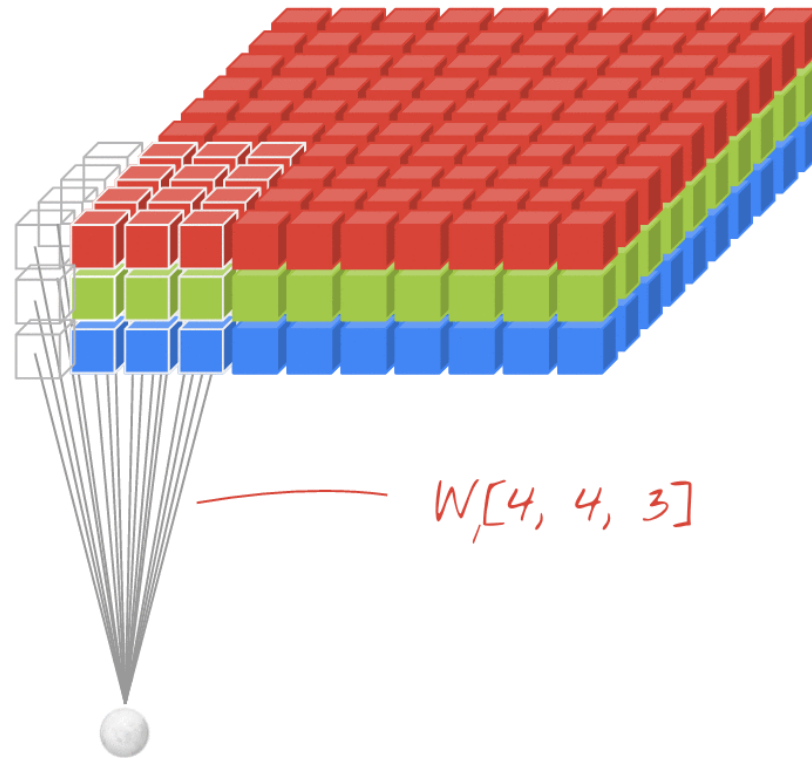


Convolutional layer

Let's say we have an image with size: $(16, 16, 3)$ and 8 kernels where each kernel is $(5, 5)$.

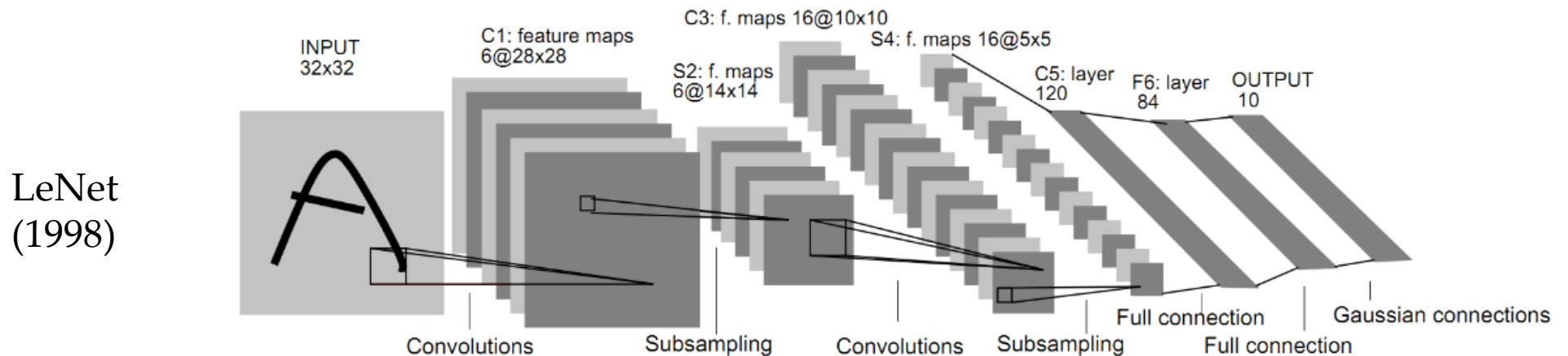
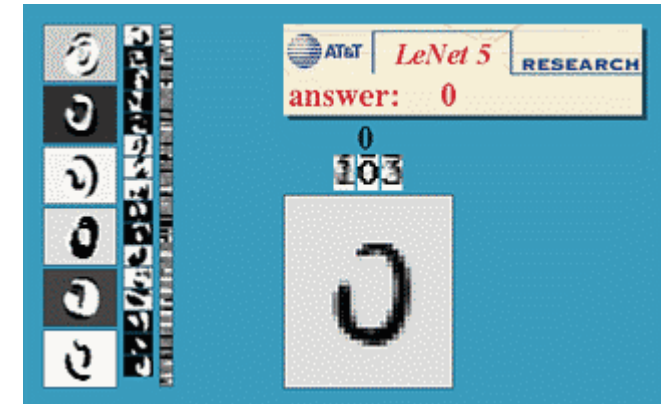


Padding and stride

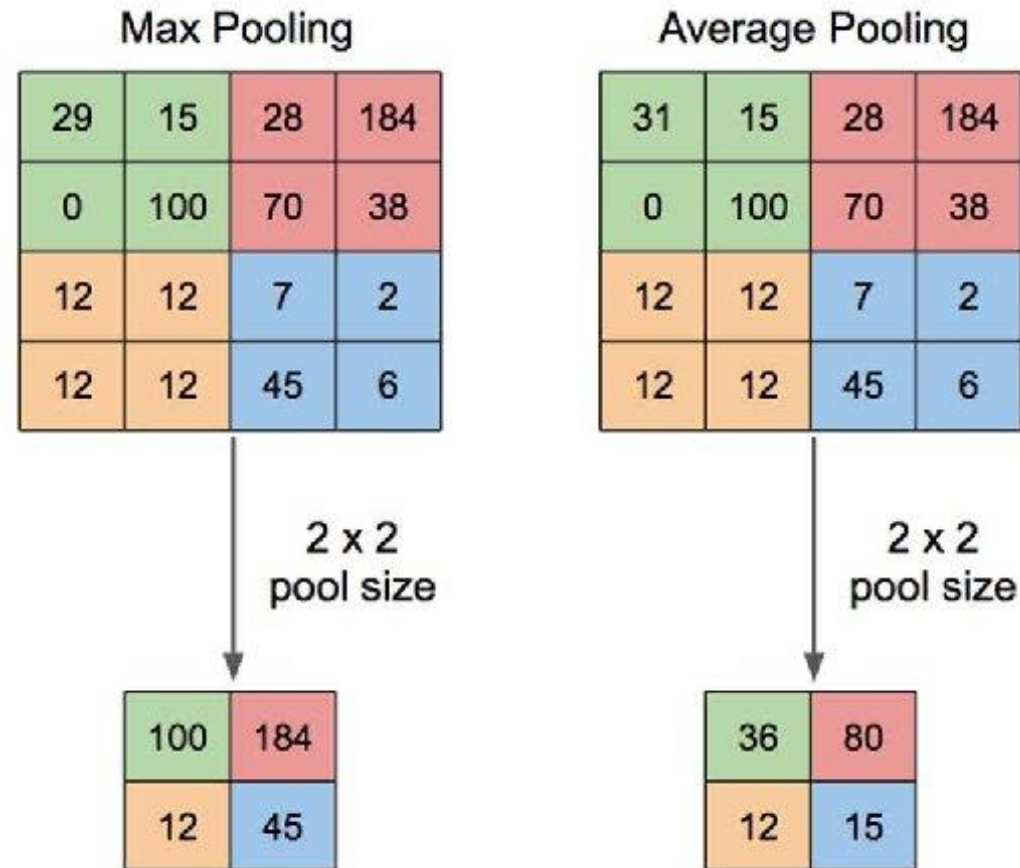


Convolutional neural networks (CNN)

- Traditionally consists of:
 - Convolutional layers
 - Nonlinearity layers
 - Pooling layers
 - Fully-connected layers

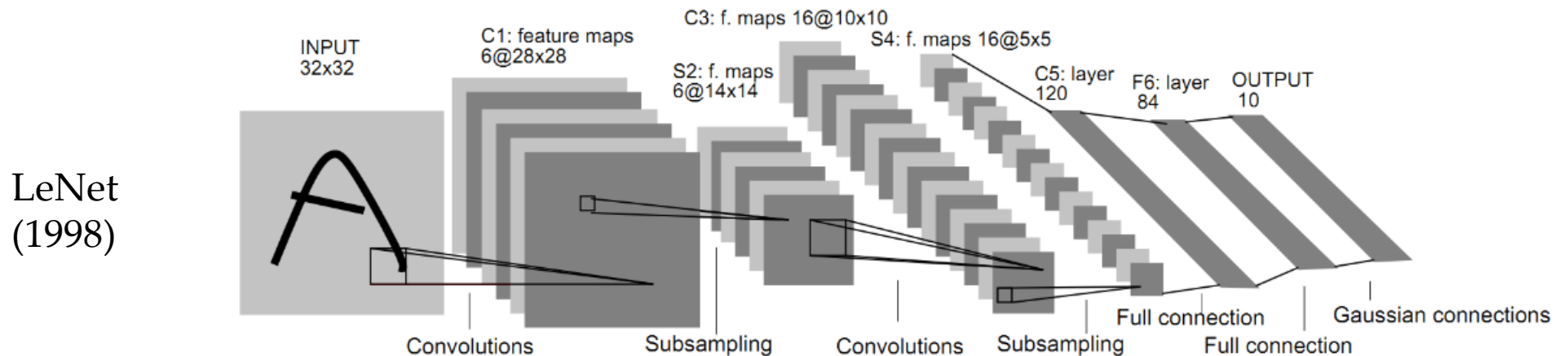
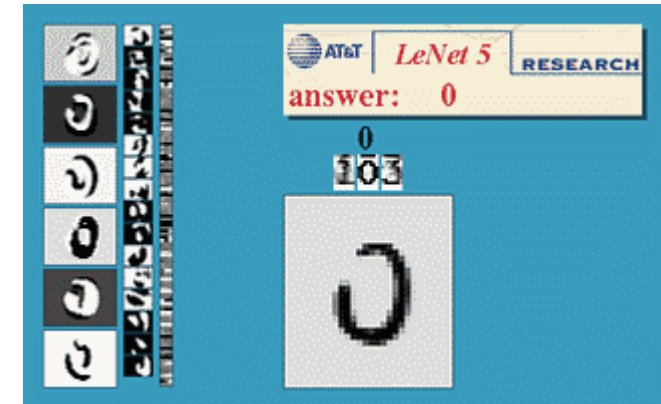


Pooling layer



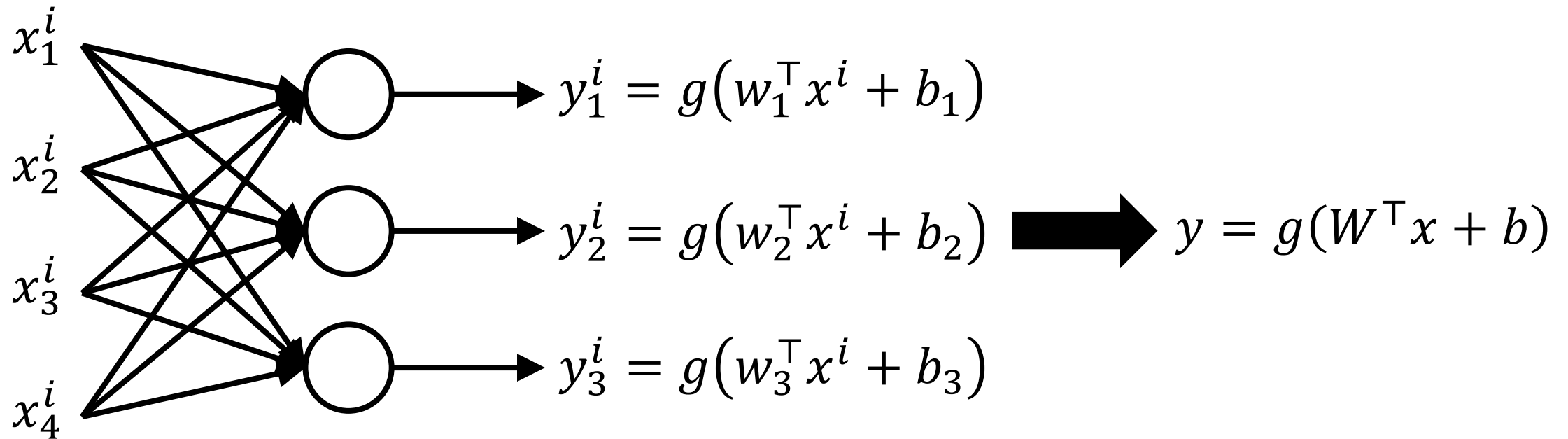
Convolutional neural networks (CNN)

- Traditionally consists of:
 - Convolutional layers
 - Nonlinearity layers
 - Pooling layers
 - Fully-connected layers



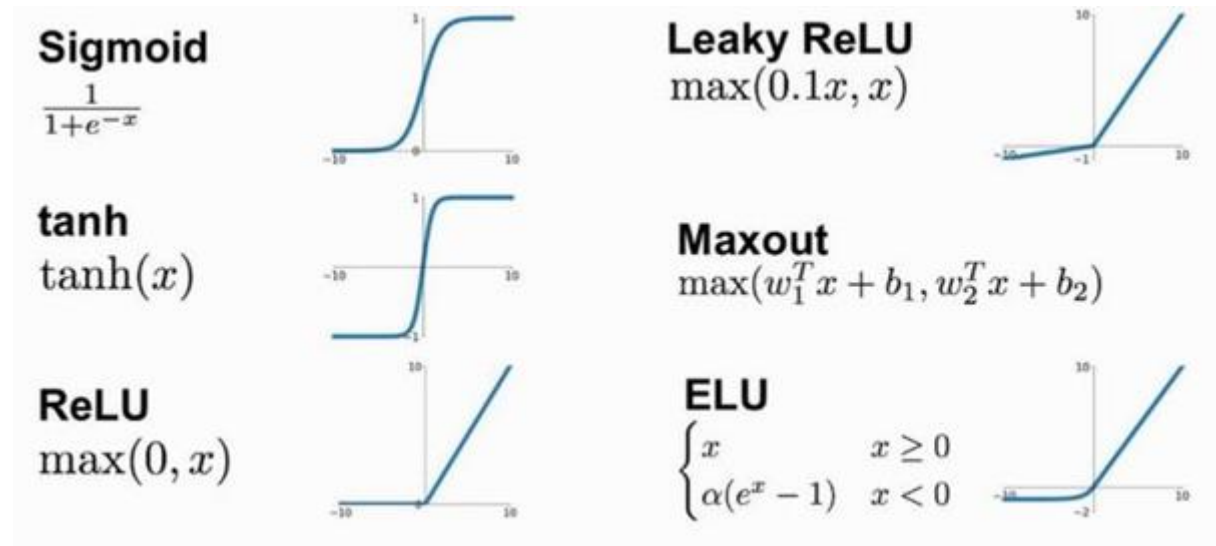
From lecture 1

A fully-connected layer



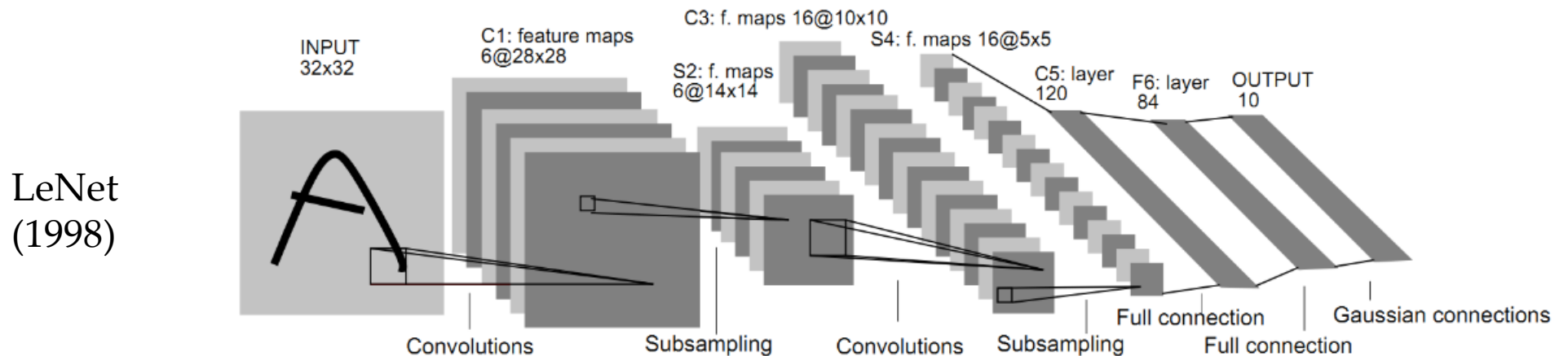
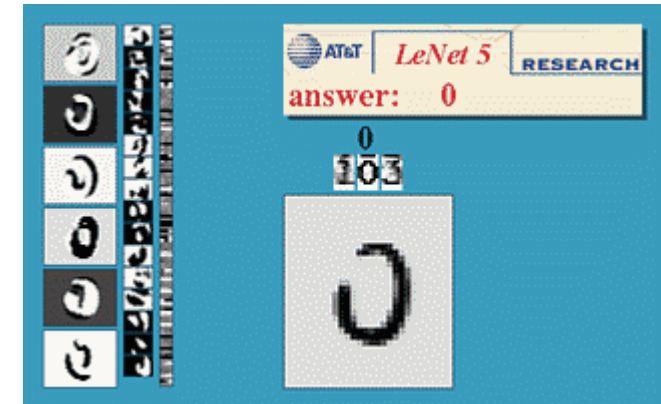
From lecture 1

Different types of nonlinearity layers:



Convolutional neural networks (CNN)

- Traditionally consists of:
 - Convolutional layers
 - Nonlinearity layers
 - Pooling layers
 - Fully-connected layers



Classification with CNNs

Classification



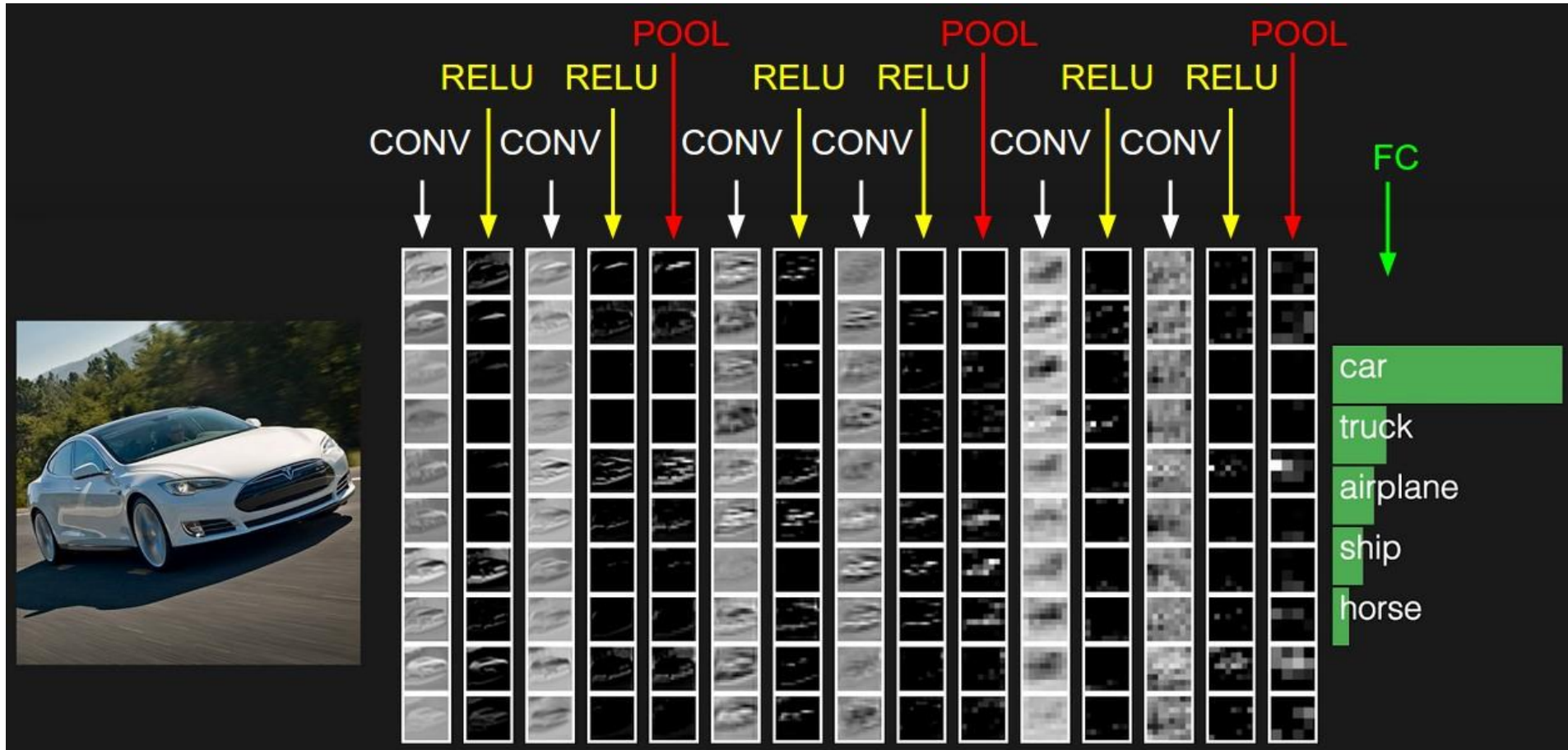
CAT

Classification via hand-crafted features

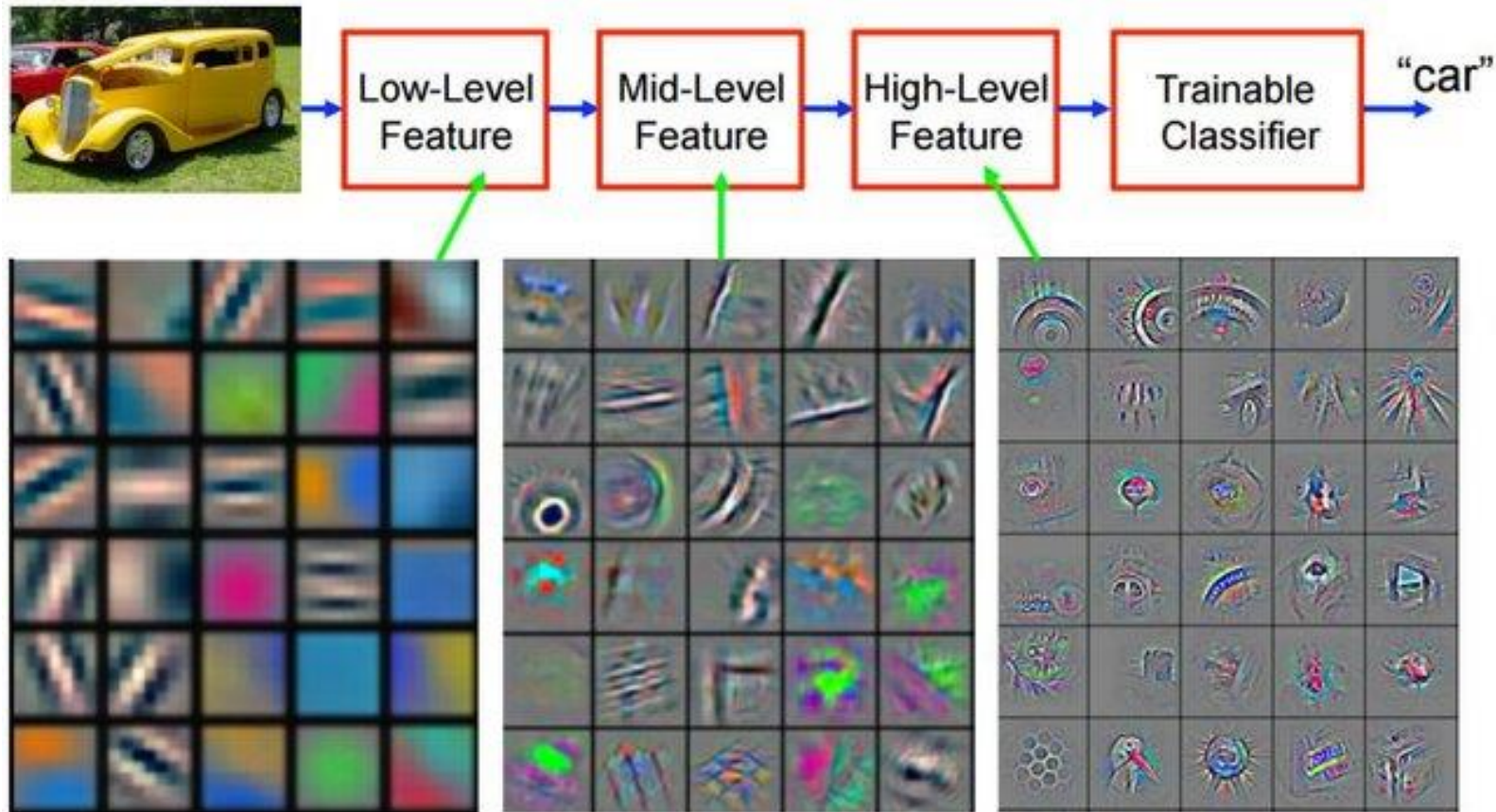
Uses image statistics and keypoints as features.

Trains a classifier over those features.

End-to-end training



Visualizing CNN features



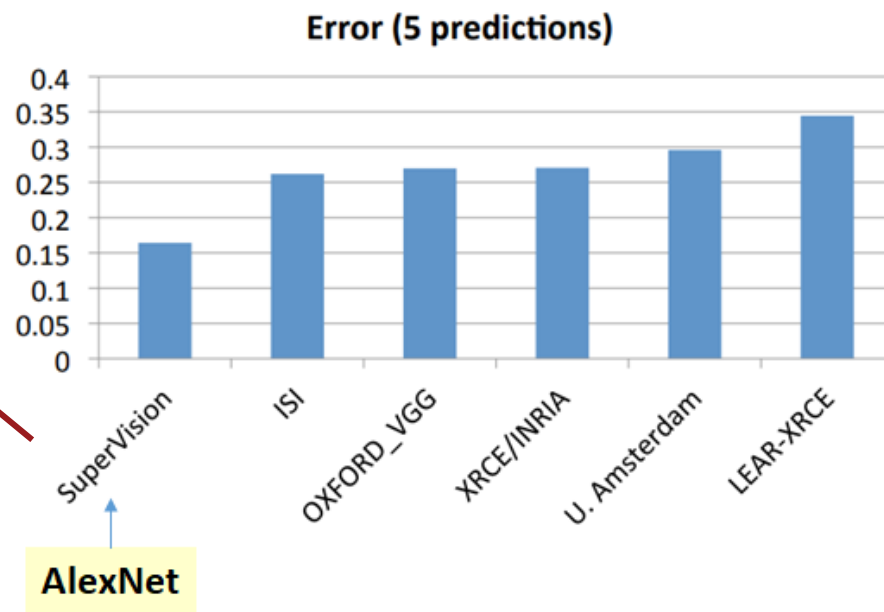
Online CNN demo

https://adamharley.com/nn_vis/

Comparison

Team name	Filename	Error (5 guesses)	Description
SuperVision	test-preds-141-146.2009-131-137-145-146.2011-145f.	0.15315	Using extra training data from ImageNet Fall 2011 release
SuperVision	test-preds-131-137-145-135-145f.txt	0.16422	Using only supplied training data
ISI	pred_FVs_wLACs_weighted.txt	0.26172	Weighted sum of scores from each classifier with SIFT+FV, LBP+FV, GIST+FV, and CSIFT+FV, respectively.
ISI	pred_FVs_weighted.txt	0.26602	Weighted sum of scores from classifiers using each FV.
ISI	pred_FVs_summed.txt	0.26646	Naive sum of scores from classifiers using each FV.
ISI	pred_FVs_wLACs_summed.txt	0.26952	Naive sum of scores from each classifier with SIFT+FV, LBP+FV, GIST+FV, and CSIFT+FV, respectively.
OXFORD_VGG	test_adhocmix_classification.txt	0.26979	Mixed selection from High-Level SVM scores and Baseline Scores, decision is performed by looking at the validation performance

Alex Krizhevsky,
Ilya Sutskever,
Geoffrey Hinton

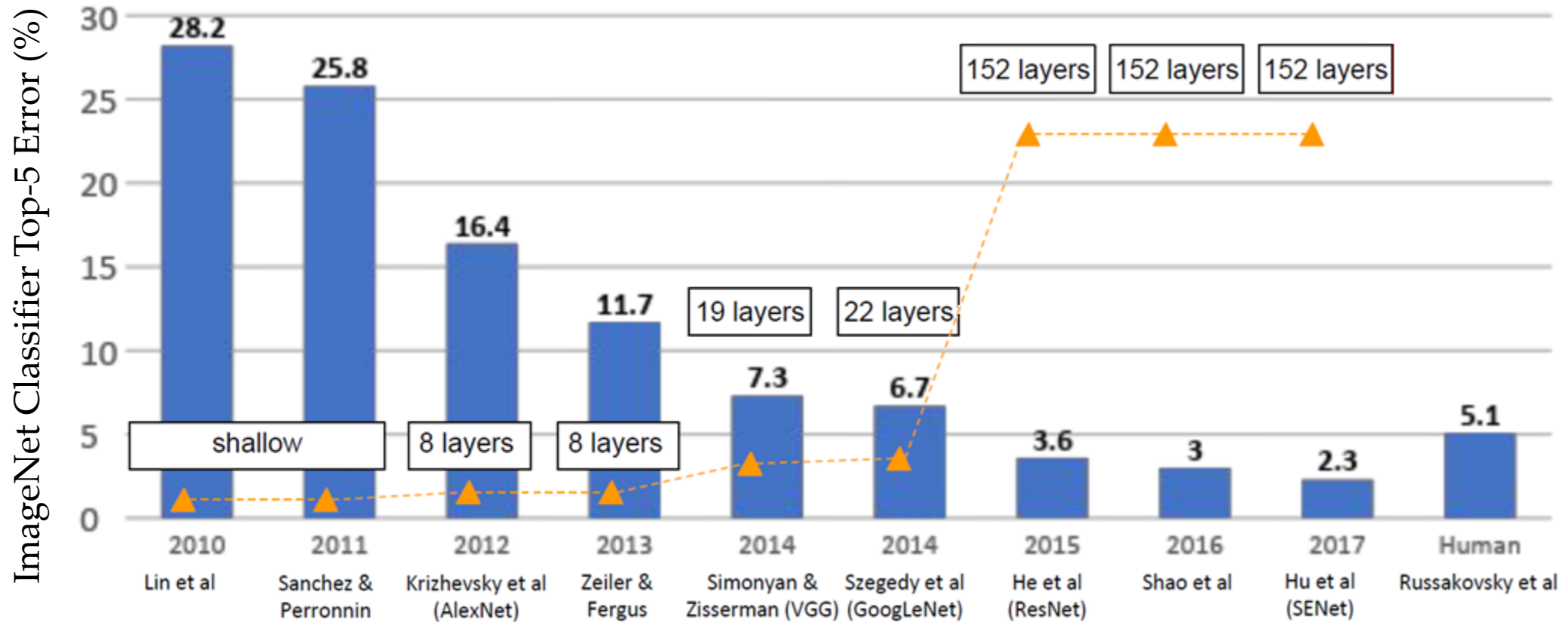


Comparison

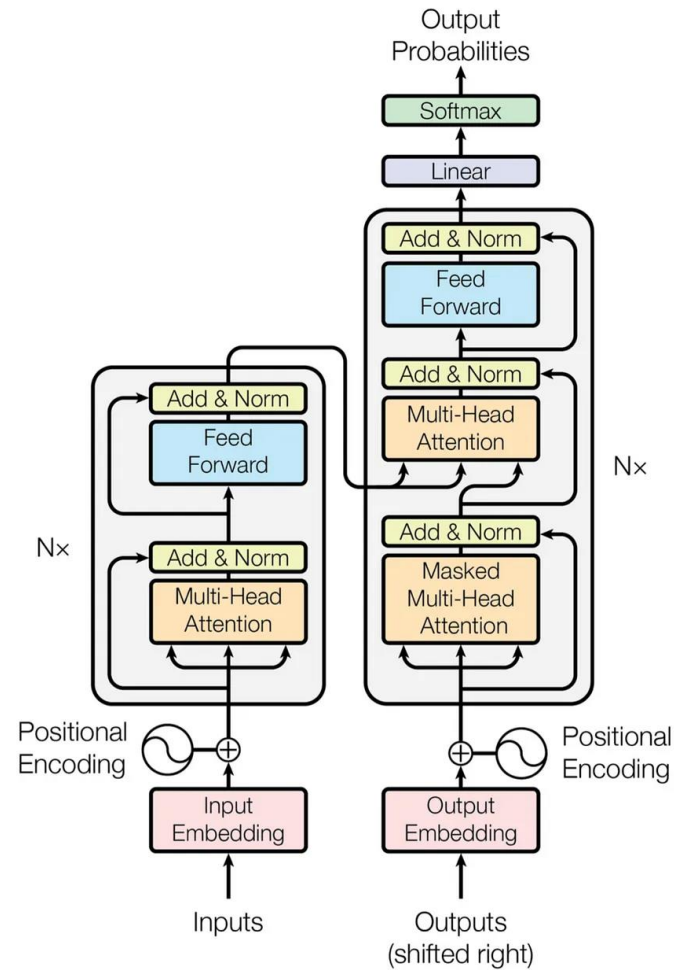
The description of SuperVision:

Our model is a large, deep convolutional neural network trained on raw RGB pixel values. The neural network, which has 60 million parameters and 650,000 neurons, consists of five convolutional layers, some of which are followed by max-pooling layers, and three globally-connected layers with a final 1000-way softmax. It was trained on two NVIDIA GPUs for about a week. To make training faster, we used non-saturating neurons and a very efficient GPU implementation of convolutional nets. To reduce overfitting in the globally-connected layers we employed hidden-unit "dropout", a recently-developed regularization method that proved to be very effective.

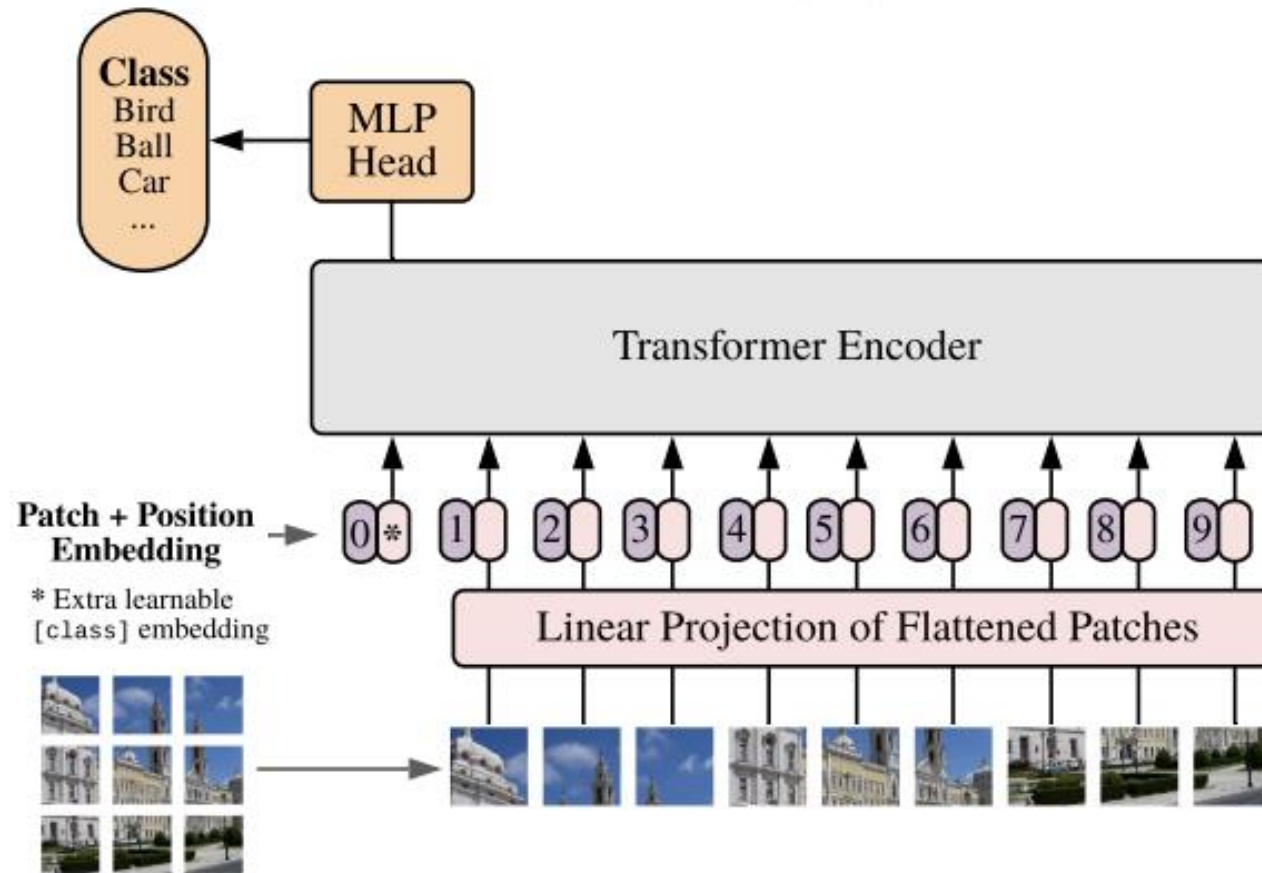
Deeper and deeper



Transformer



Vision transformers (ViT)



Robotics applications of vision

Classification



CAT

Robotics applications of vision

Classification



CAT

**Classification
+ Localization**

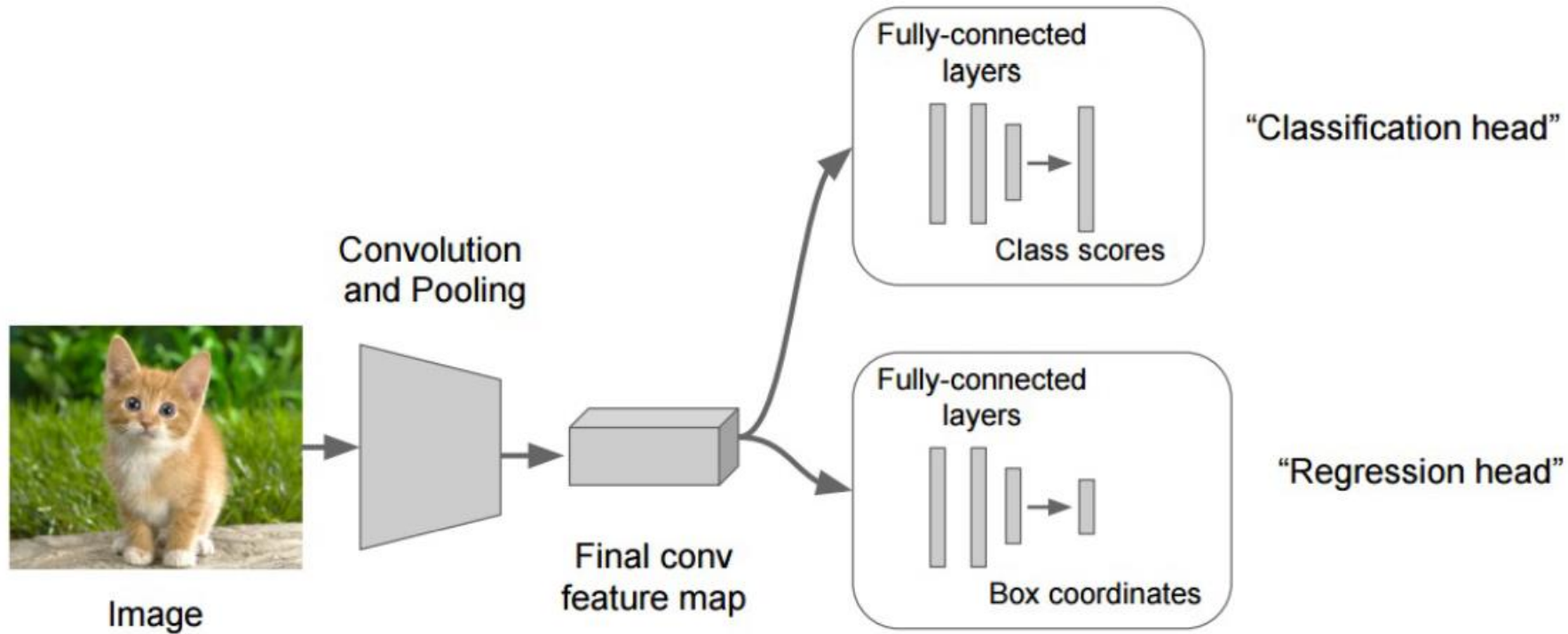


CAT

Classification + Localization

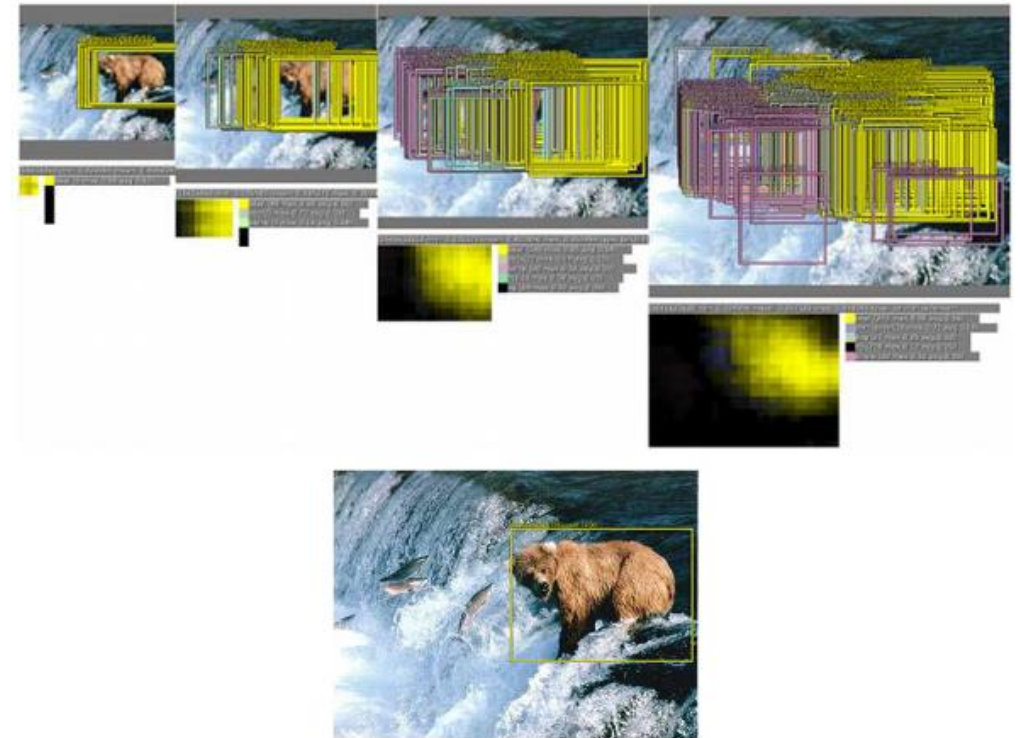
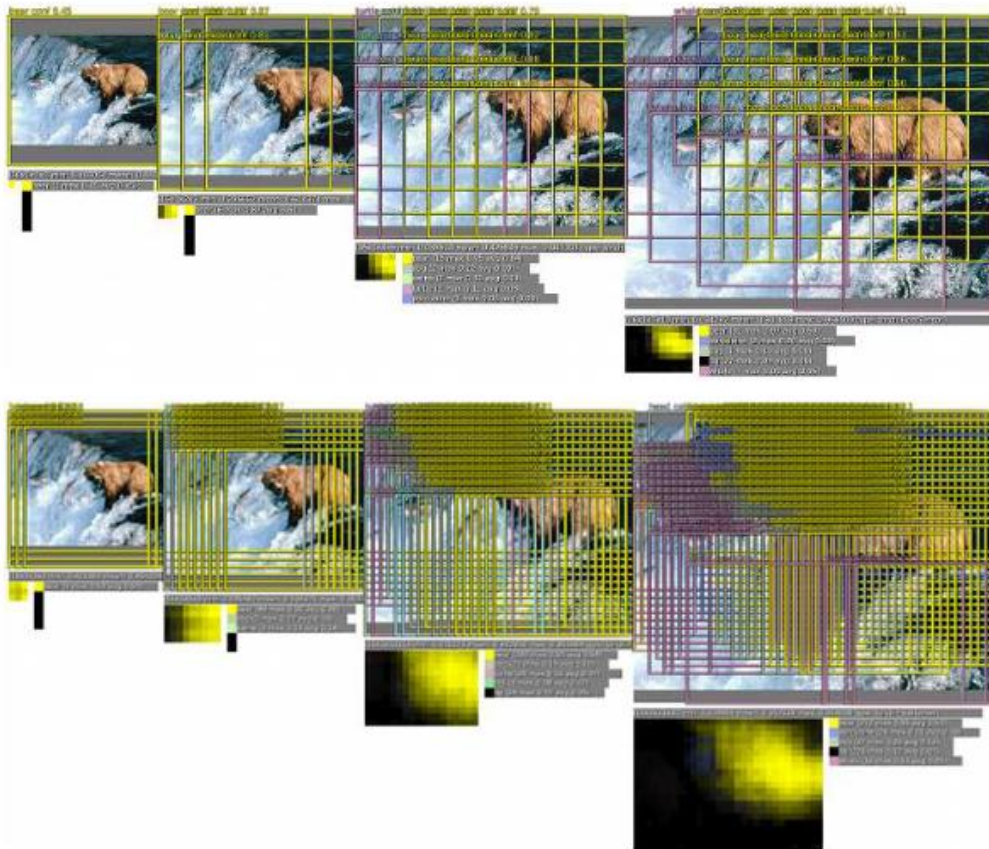
Do not just output the image class.

Output the class and 4 numbers, one for each corner of the bounding box.



Classification + Localization

Or just use a classifier multiple times.



Robotics applications of vision

Classification



CAT

**Classification
+ Localization**



CAT

Single object

Robotics applications of vision

Classification



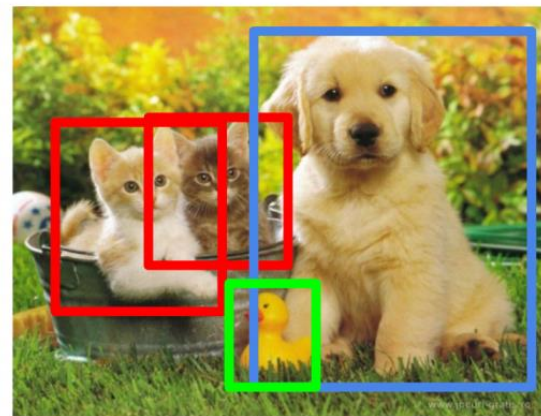
CAT

**Classification
+ Localization**



CAT

Object Detection



CAT, DOG, DUCK

Single object

Robotics applications of vision

Classification



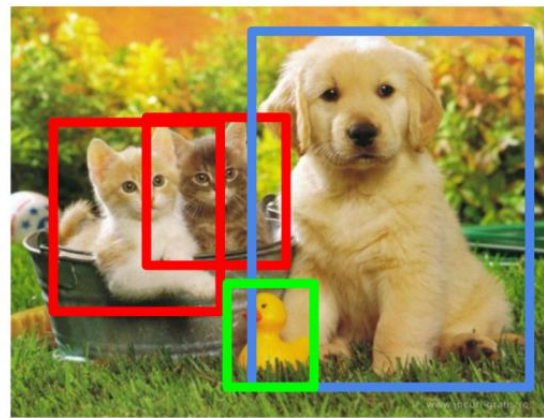
CAT

**Classification
+ Localization**



CAT

Object Detection



CAT, DOG, DUCK

**Instance
Segmentation**

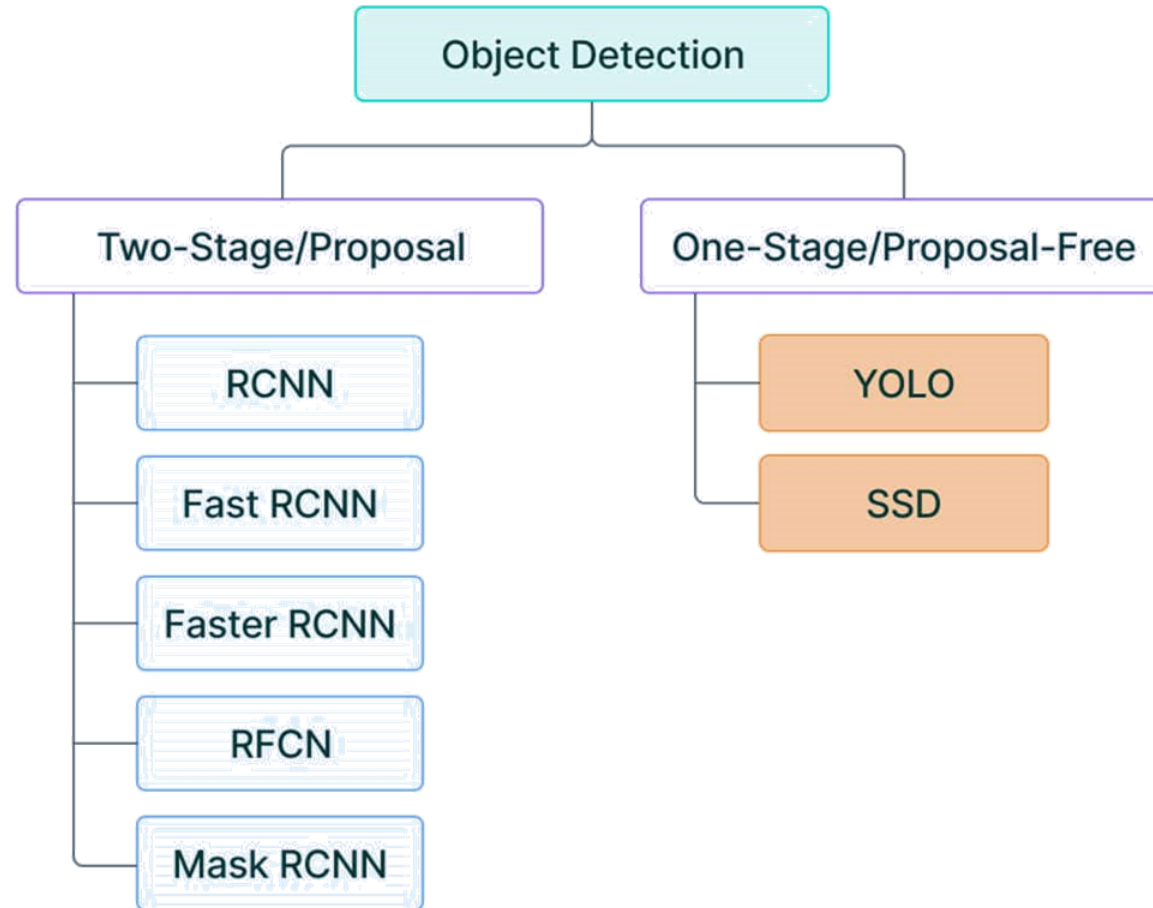


CAT, DOG, DUCK

Single object

Multiple objects

Object detection



Metric for object detection

$$IoU = \frac{\text{Area of Overlap}}{\text{Area of Union}} =$$


Object detection with YOLO

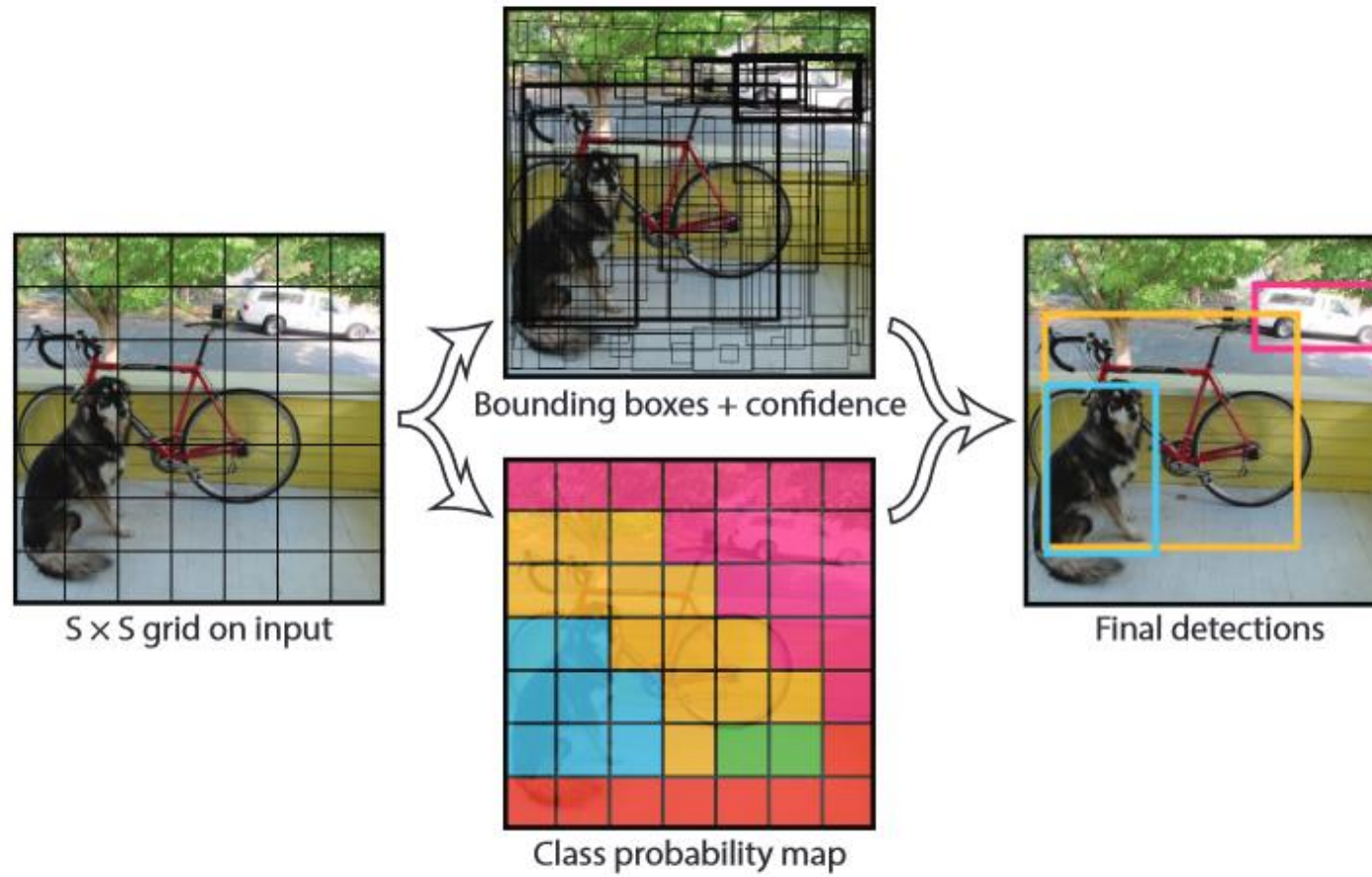
Divide image into $S \times S$ grid. For each cell, predict:

- B boxes with 4 coordinates and 1 confidence score each, and
- C class scores.

This gives a tensor of $S \times S \times (5B + C)$.

Apply non-maximum suppression to finalize.

Object detection with YOLO



Object detection in robotics

Speed is crucial – we need to detect, track, and classify many objects with high frequency.

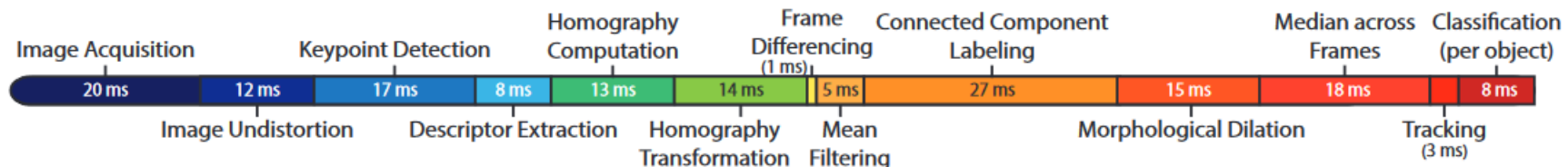
Real-Time Detection, Tracking and Classification of Multiple Moving Objects in UAV Videos

Hüseyin C. Baykara*, Erdem Bıyık*, Gamze Gül*, Deniz Onural*, Ahmet S. Öztürk*, İlkey Yıldız*

International Conference on Tools with Artificial Intelligence (ICTAI), November 2017

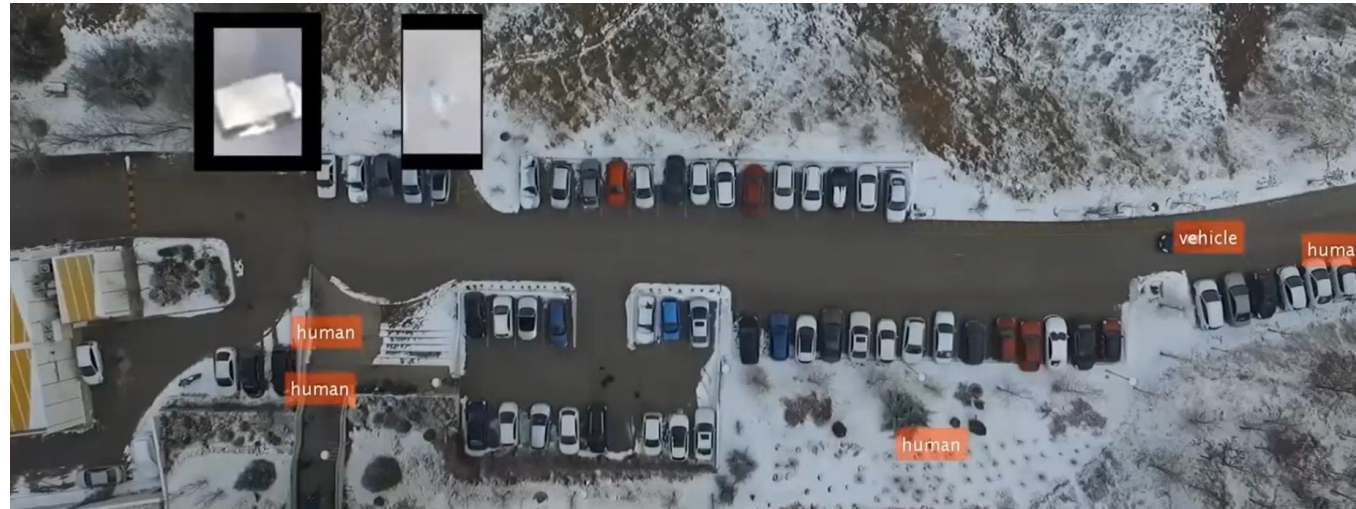


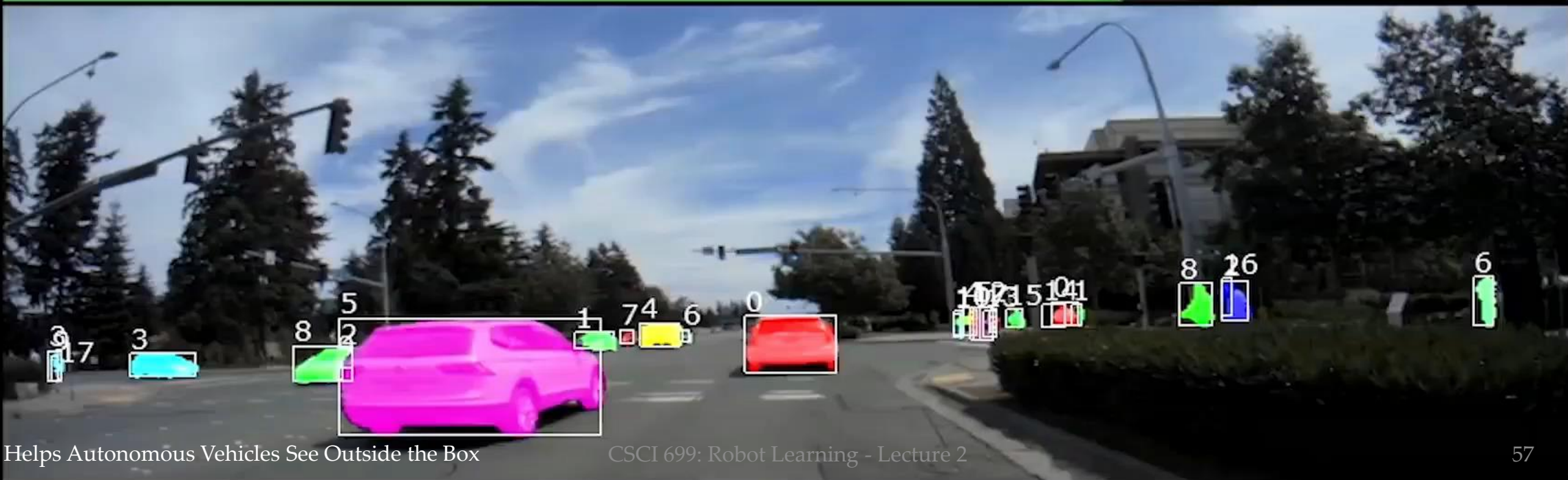
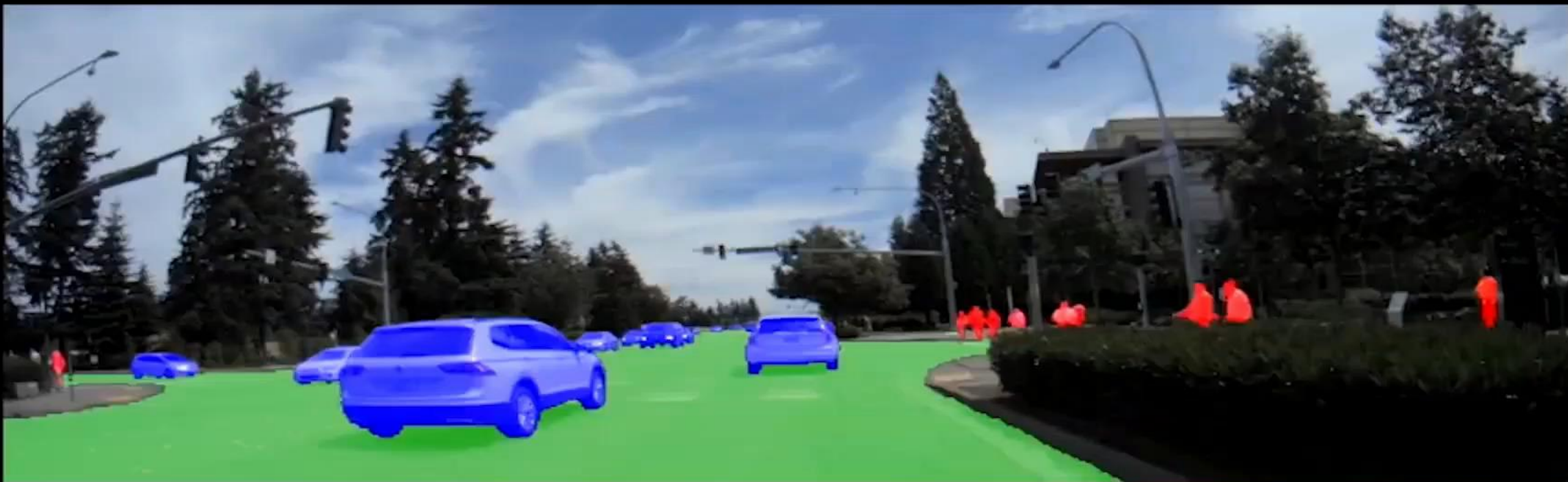
* denotes equal contribution.



Object detection in robotics

Speed is crucial – we need to detect, track, and classify many objects with high frequency.





Today

- Basics of computer vision for robotics
- Representation learning

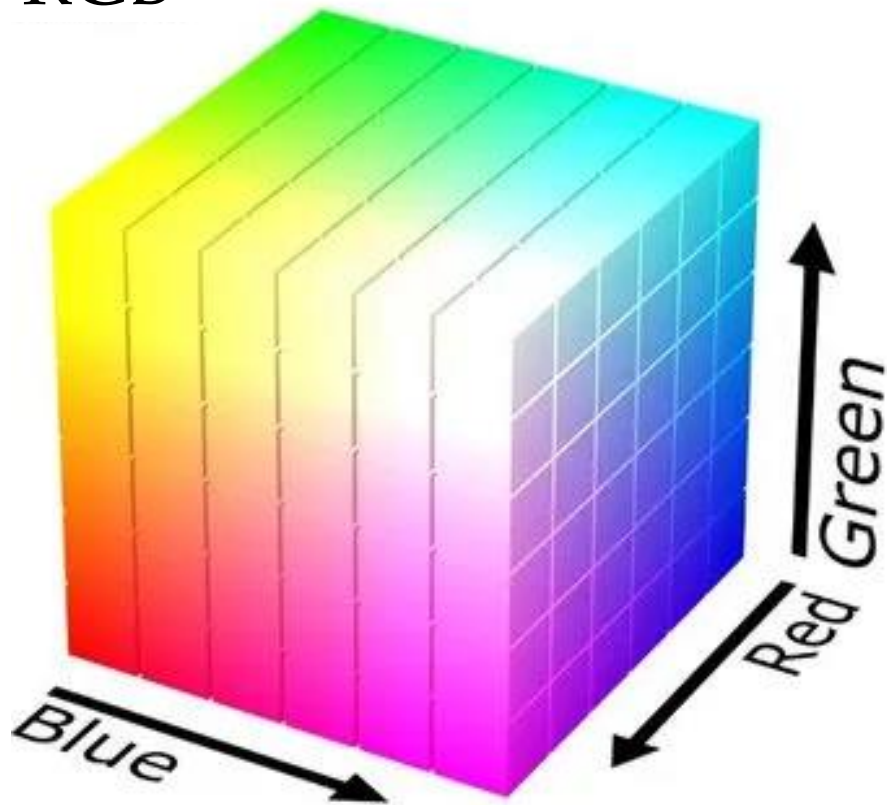
Representation learning

A good representation:

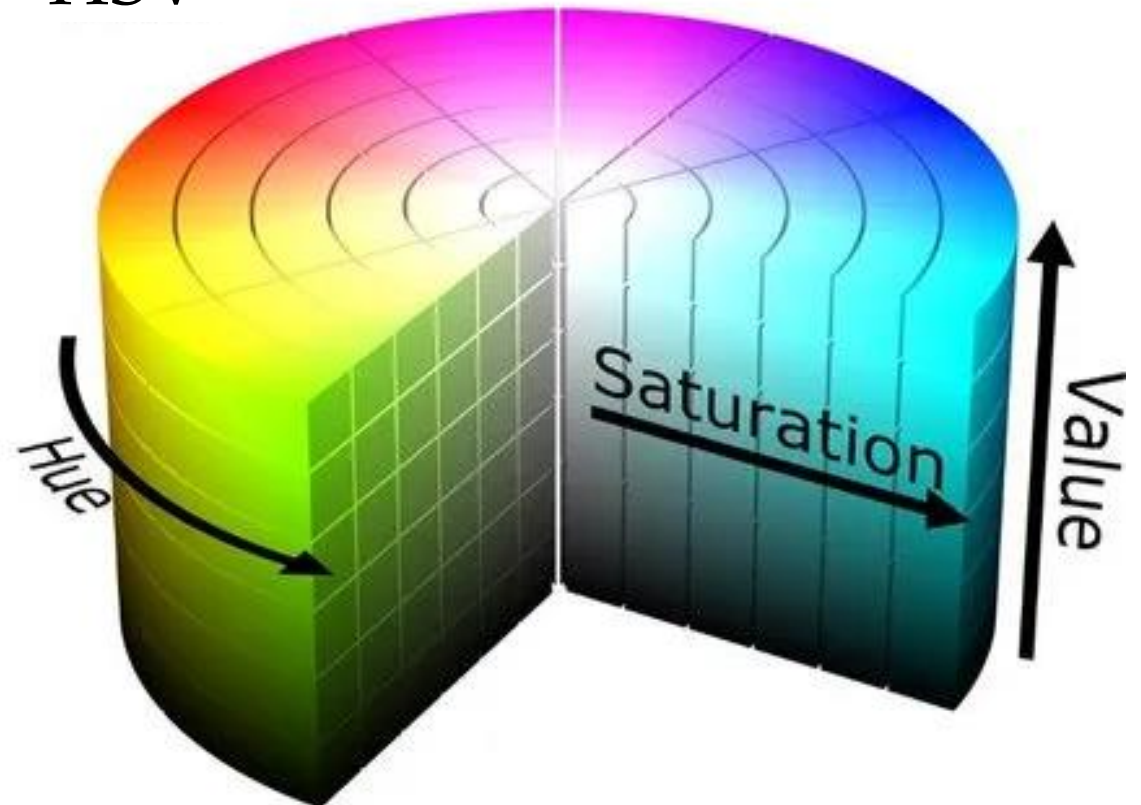
- contains the aspects of the raw data that are important for the downstream task
- is usually compact
- is ideally (but rarely) interpretable

Examples of representations

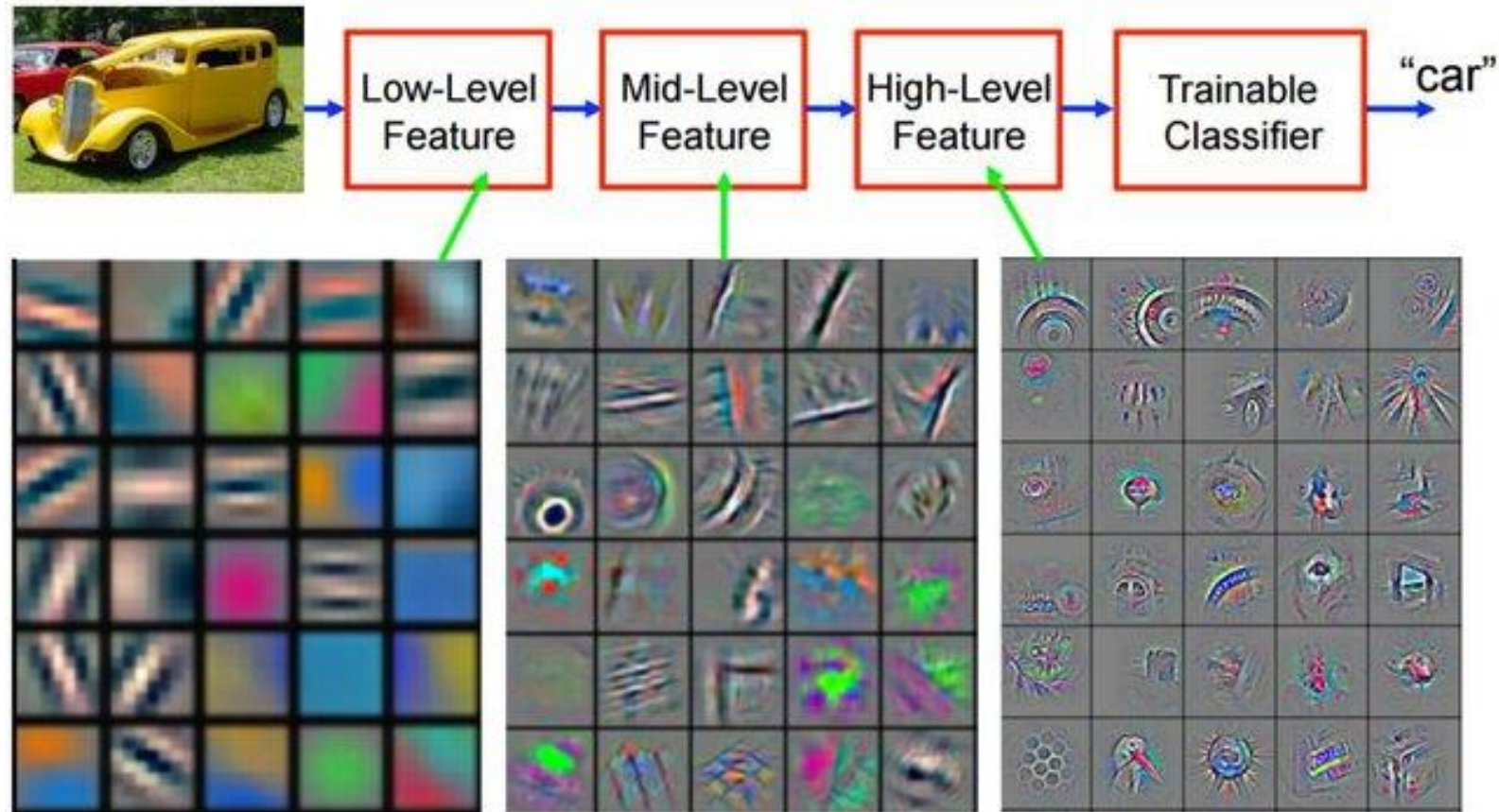
RGB



HSV



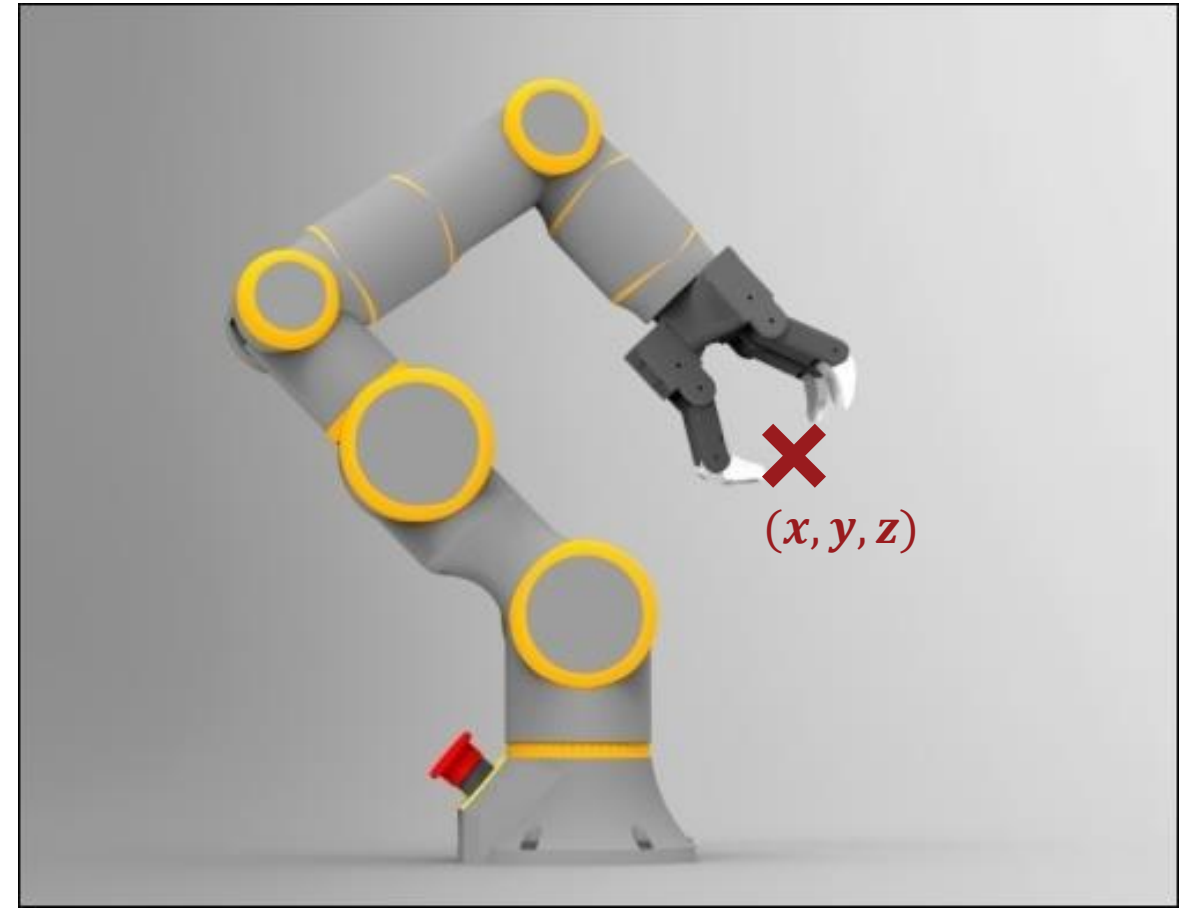
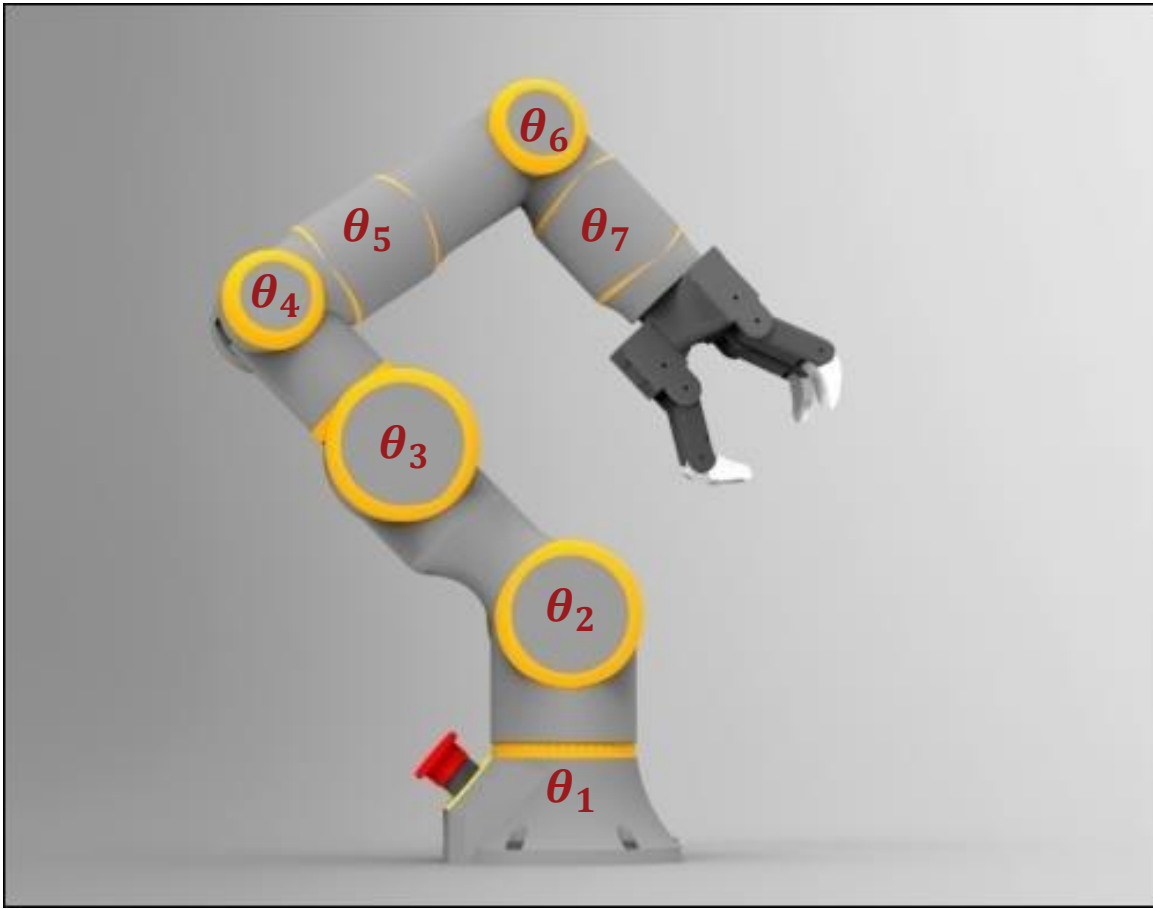
Examples of representations



Examples of representations



Examples of representations



How do we learn representations?

So many methods that it could be a course on its own.

Dynamics of Representation Learning

30117D O48 4.0 Lecture 12:00-1:50pm Mon, Wed 23 of 28  Greg Ver Steeg

Advanced Topics in Representation Learning for NLP

30111D O48 4.0 Lecture 2:00-5:20pm Tuesday 16 of 18  Xiang Ren

Representation Learning: Theory and Practice

30220D O48 4.0 Lecture 10:00-11:50am Mon, Wed 51 of 52  Aram Galstyan,
 Greg Ver Steeg

How do we learn representations?

So many methods that it could be a course on its own.

We will cover only two:

- Autoencoders
- Variational autoencoders

Principal component analysis (PCA)

PCA is a special case of autoencoders.

Raw data:

$$X \in \mathbb{R}^{n \times d}$$

Top k eigenvectors:

$$V \in \mathbb{R}^{d \times k}$$

Representation:

$$Z = XV \in \mathbb{R}^{n \times k}$$

Reconstruction:

$$\hat{X} = XVV^T \in \mathbb{R}^{n \times d}$$

Principal component analysis (PCA)

PCA is a special case of autoencoders.

Raw data:

$$X \in \mathbb{R}^{n \times d}$$

Top k eigenvectors:

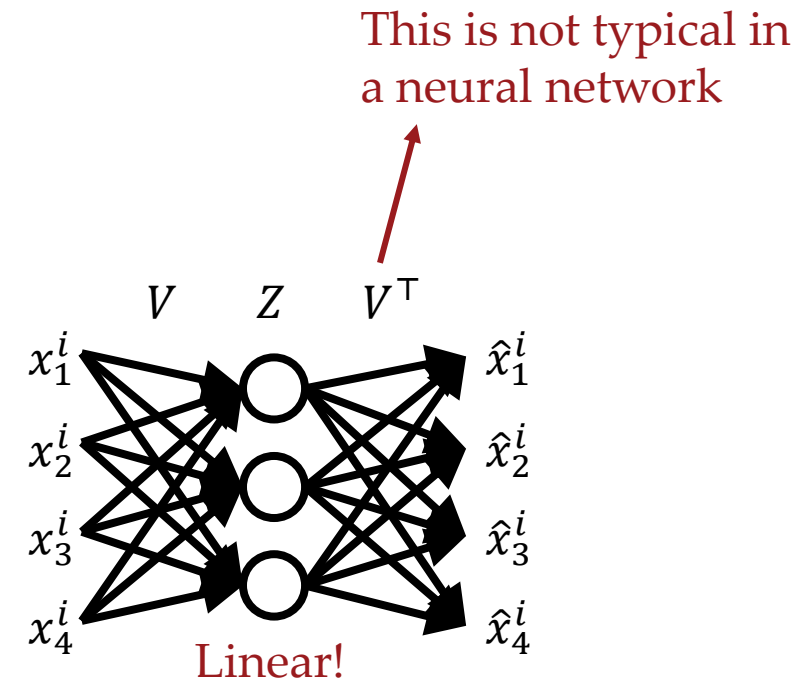
$$V \in \mathbb{R}^{d \times k}$$

Representation:

$$Z = XV \in \mathbb{R}^{n \times k}$$

Reconstruction:

$$\hat{X} = XVV^T \in \mathbb{R}^{n \times d}$$

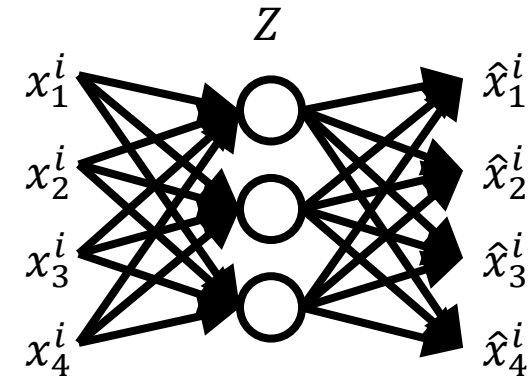


Principal component analysis (PCA)

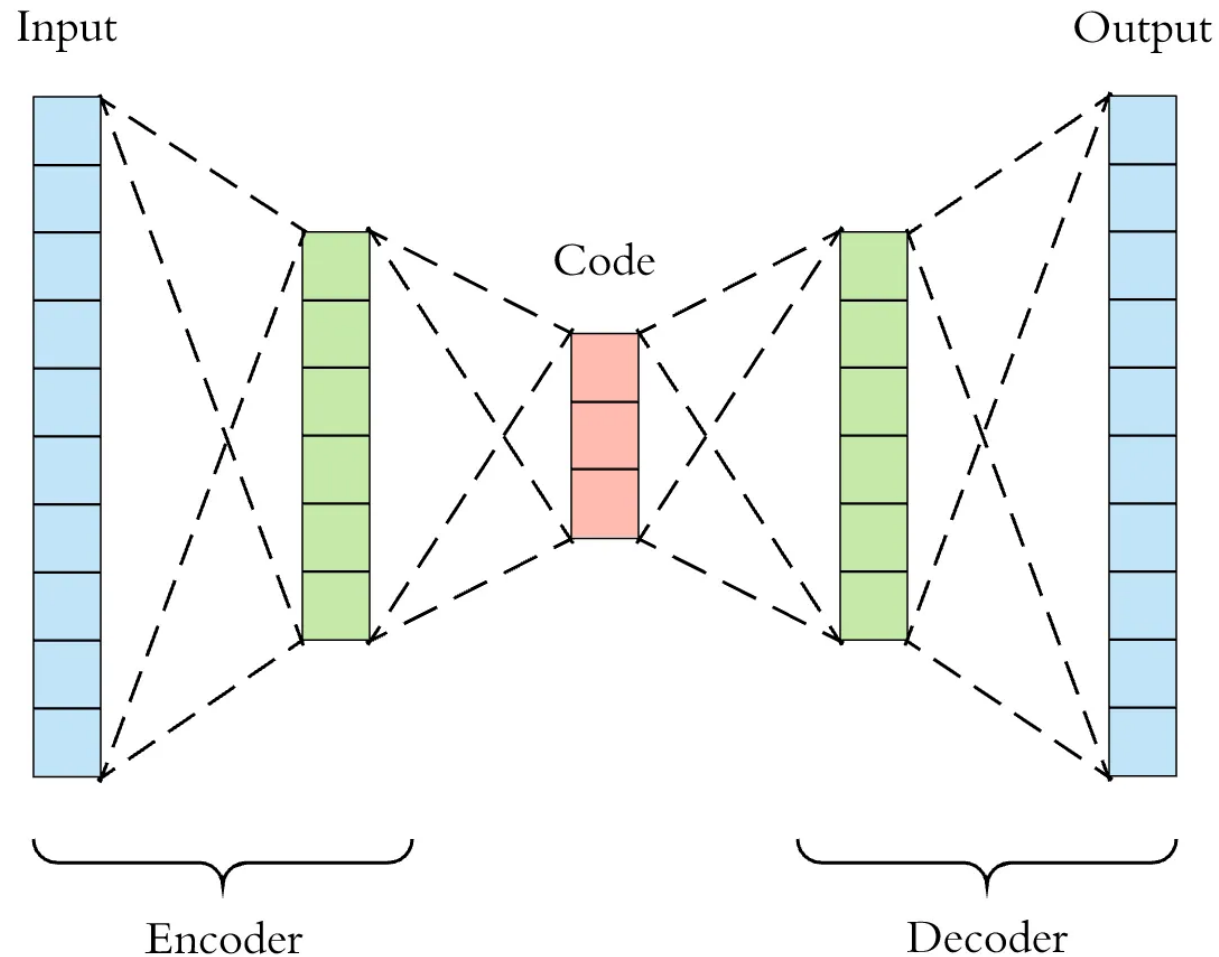
PCA is a special case of autoencoders.

1. Add nonlinearity
2. Remove the dependence between layers
3. Possibly add more layers

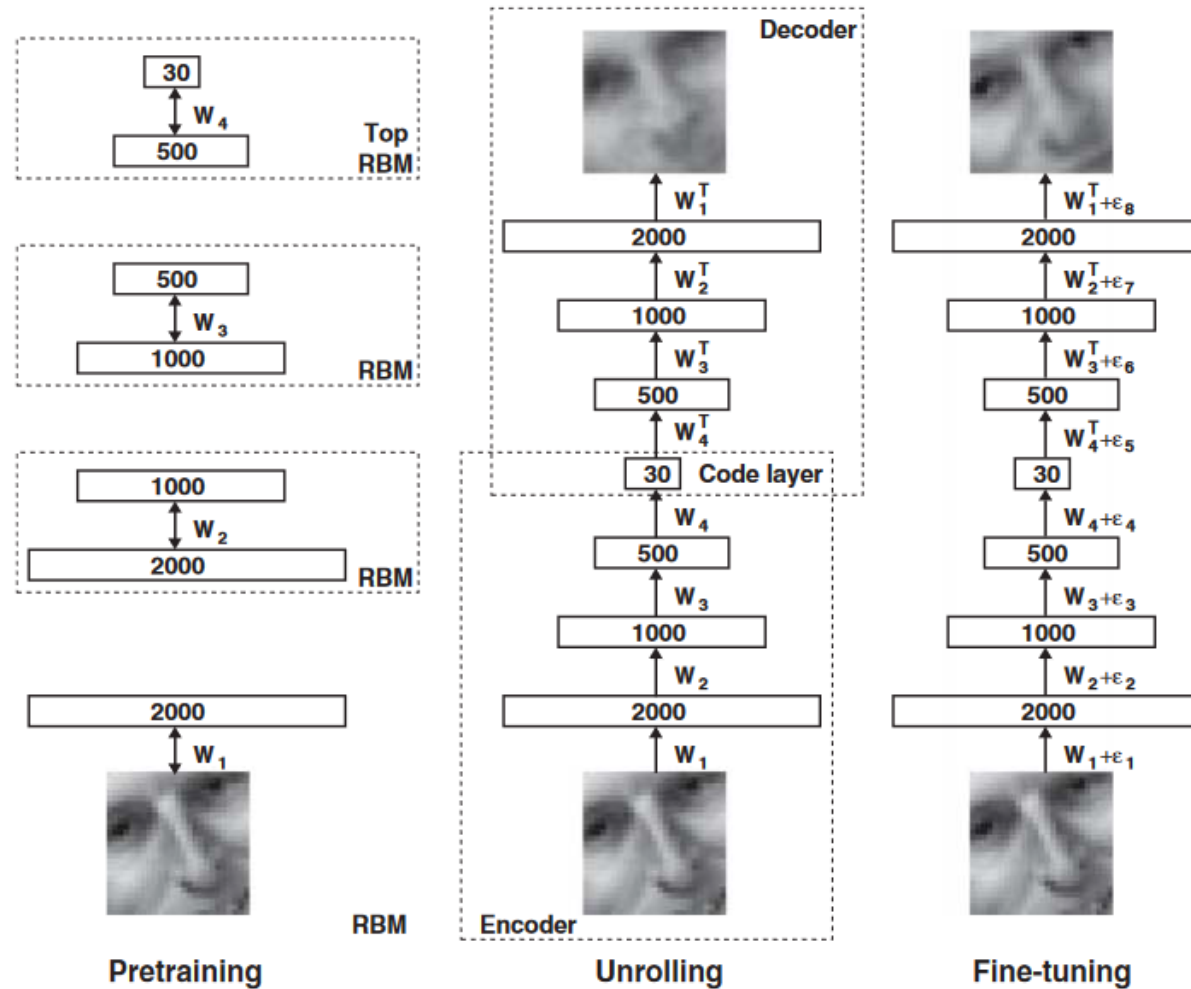
Now you have a more powerful autoencoder.



Autoencoders

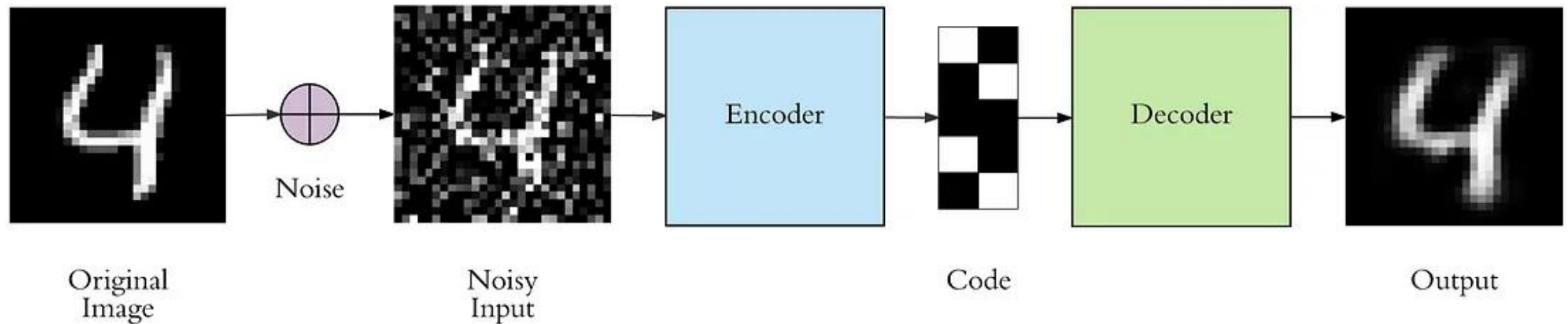


Autoencoders



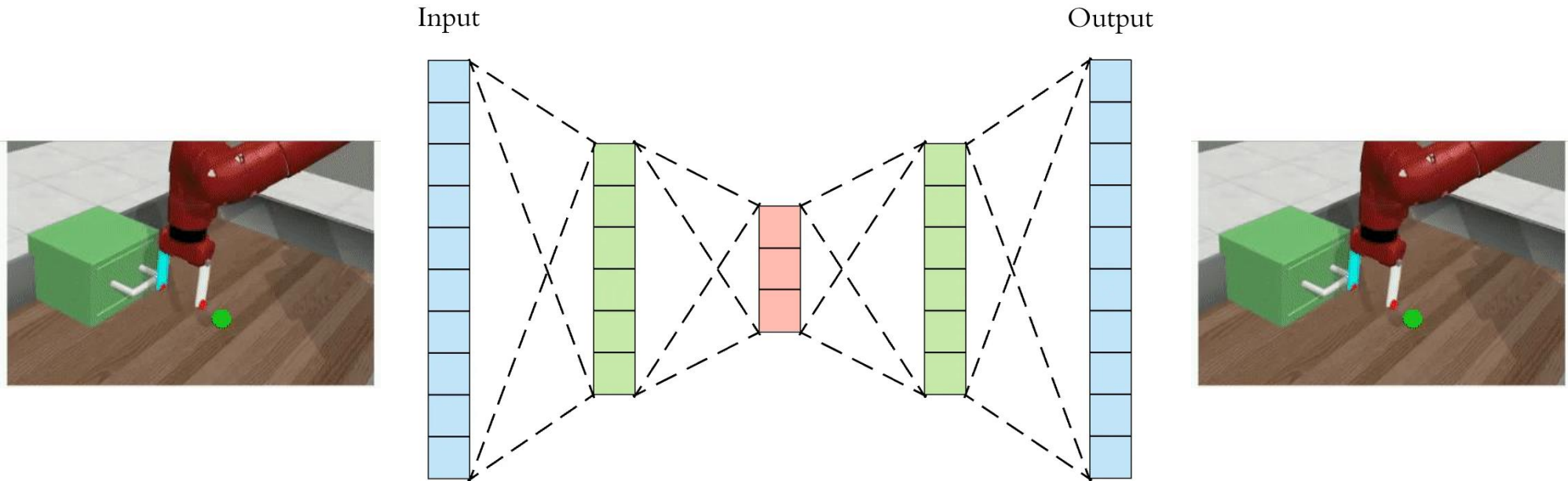
Applications of autoencoders

- Denoising



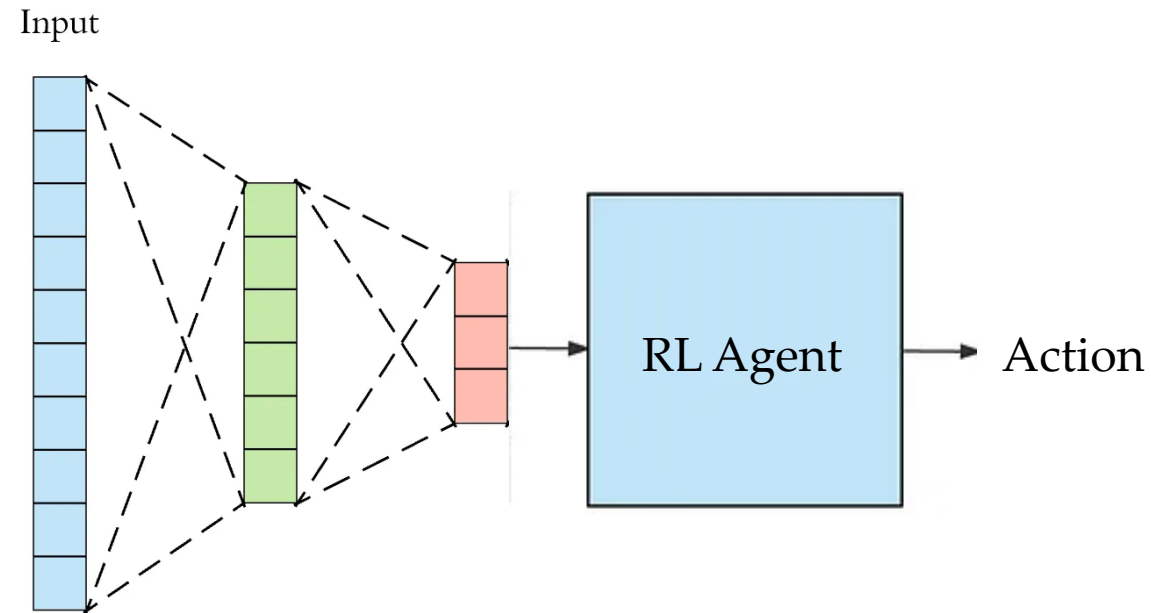
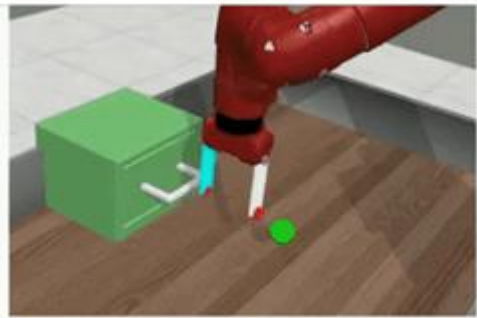
Applications of autoencoders

- Reinforcement learning



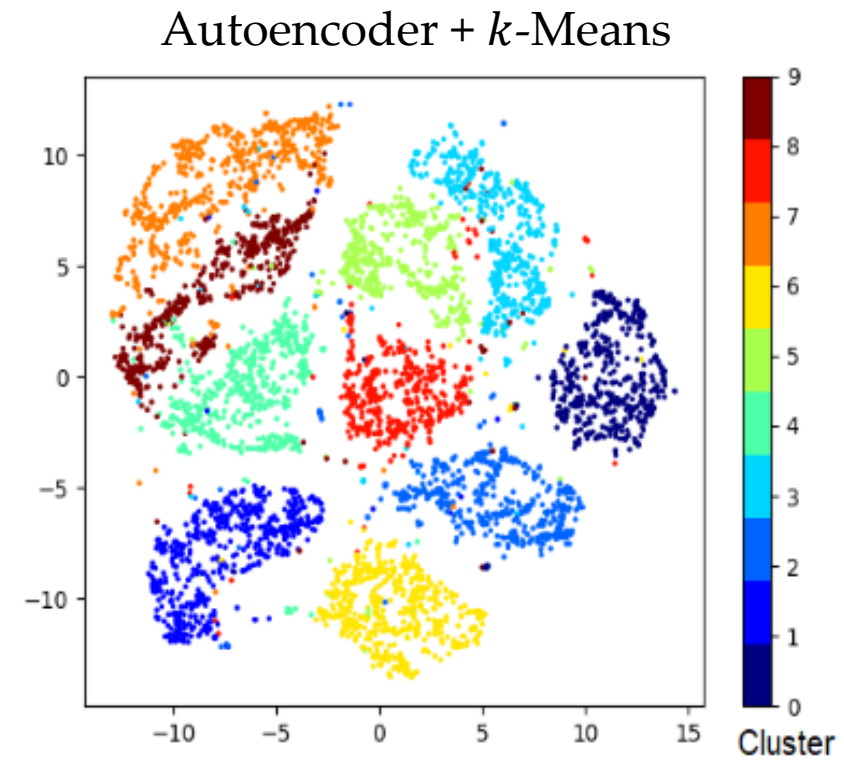
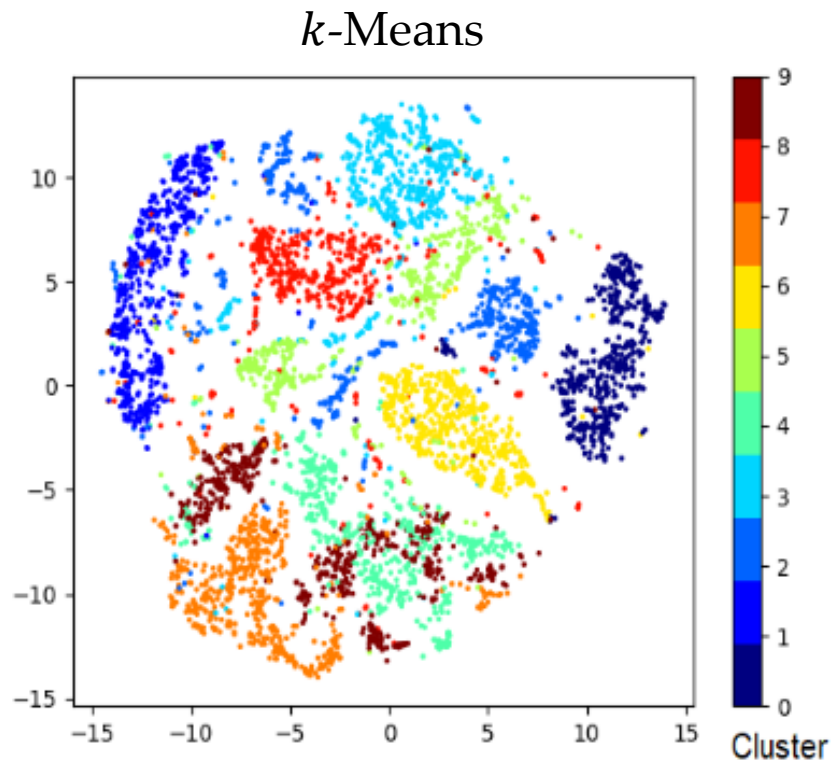
Applications of autoencoders

- Reinforcement learning



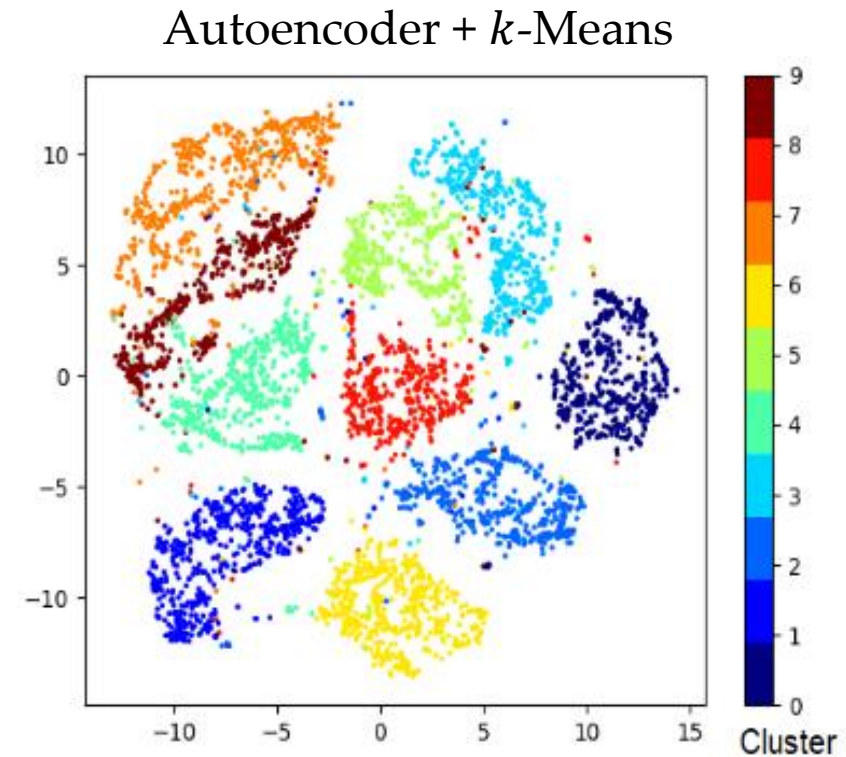
Applications of autoencoders

- Clustering



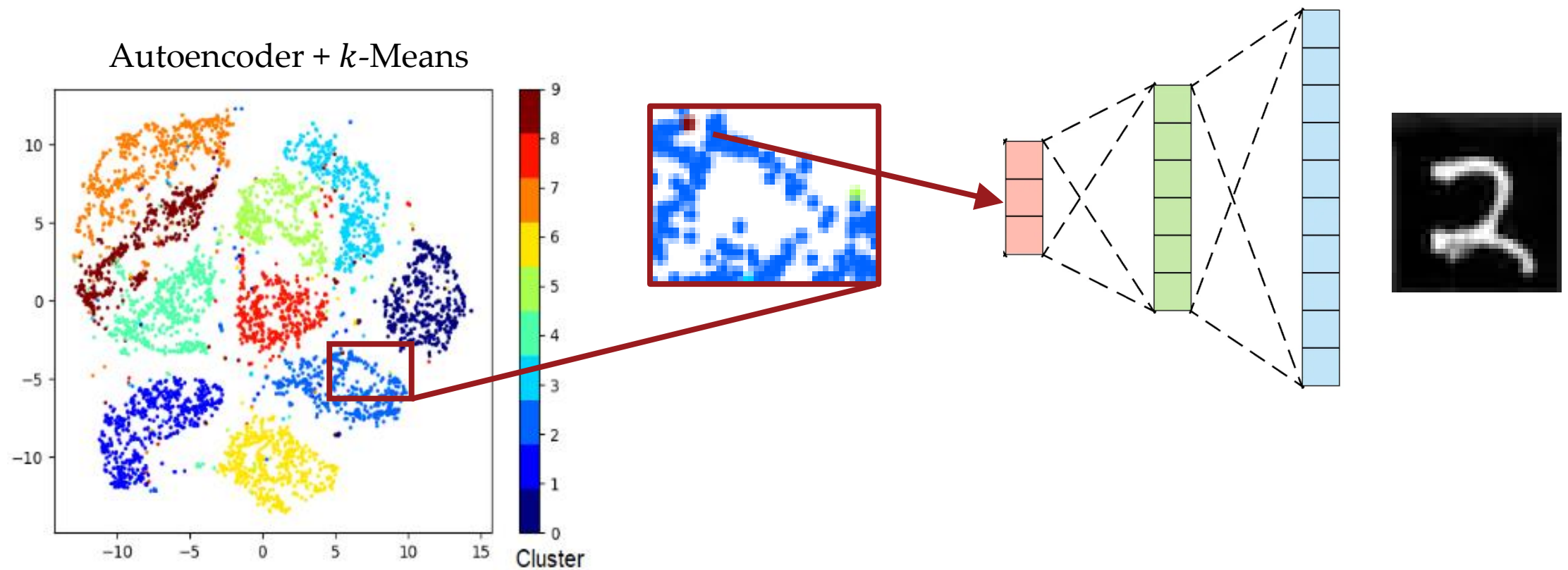
Applications of autoencoders

- Generate new data??



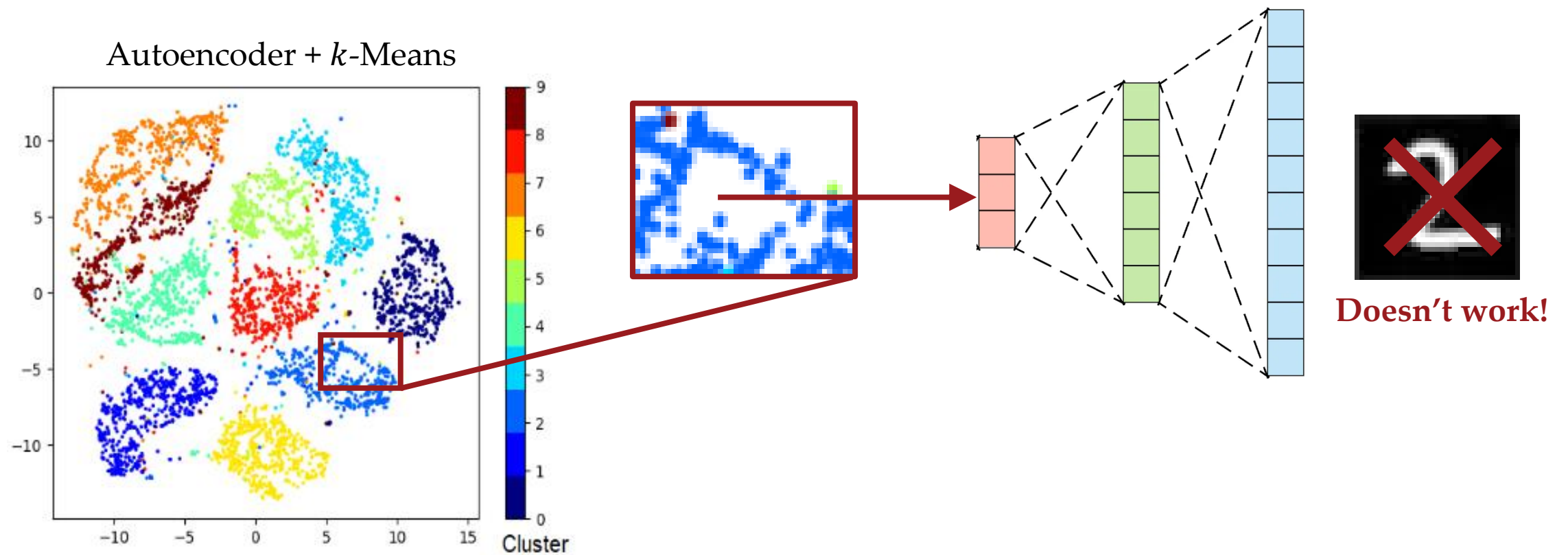
Applications of autoencoders

- Generate new data??



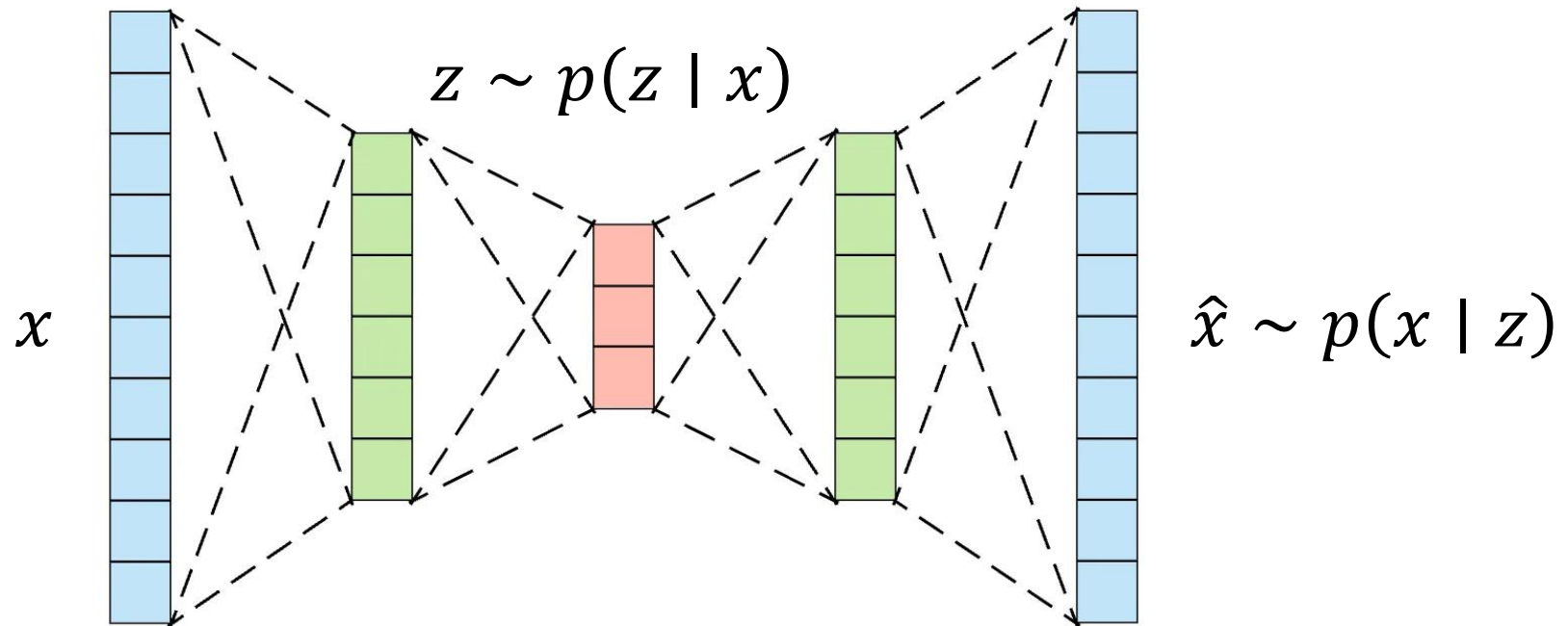
Applications of autoencoders

- Generate new data??



Variational autoencoders (VAE)

Same as autoencoders, but everything is a distribution now.



Variational autoencoders (VAE)

How does a neural network output a distribution?

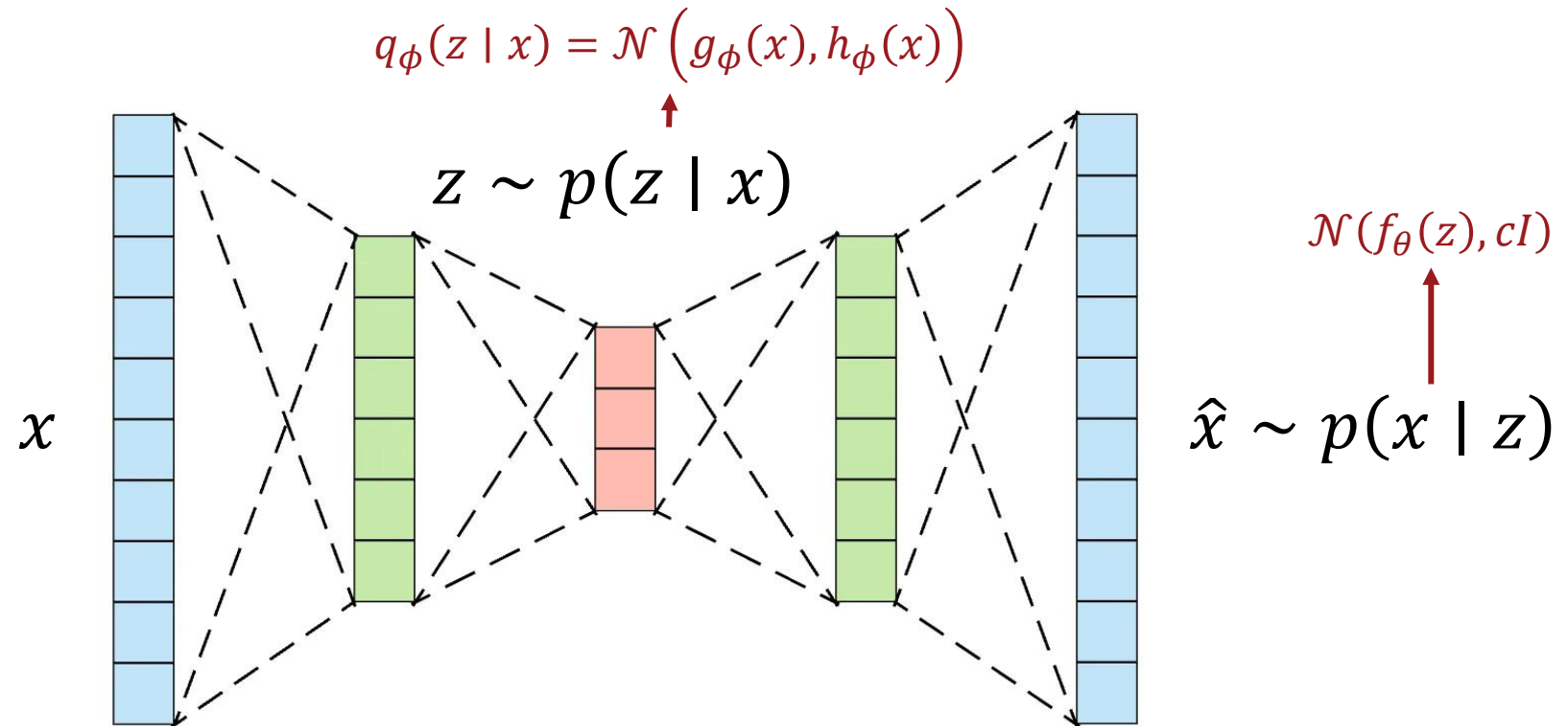
We assume a parameterized distribution, and the network outputs the parameters.



Most commonly a Gaussian distribution. Then, the network outputs the mean vector and the covariance matrix.

Variational autoencoders (VAE)

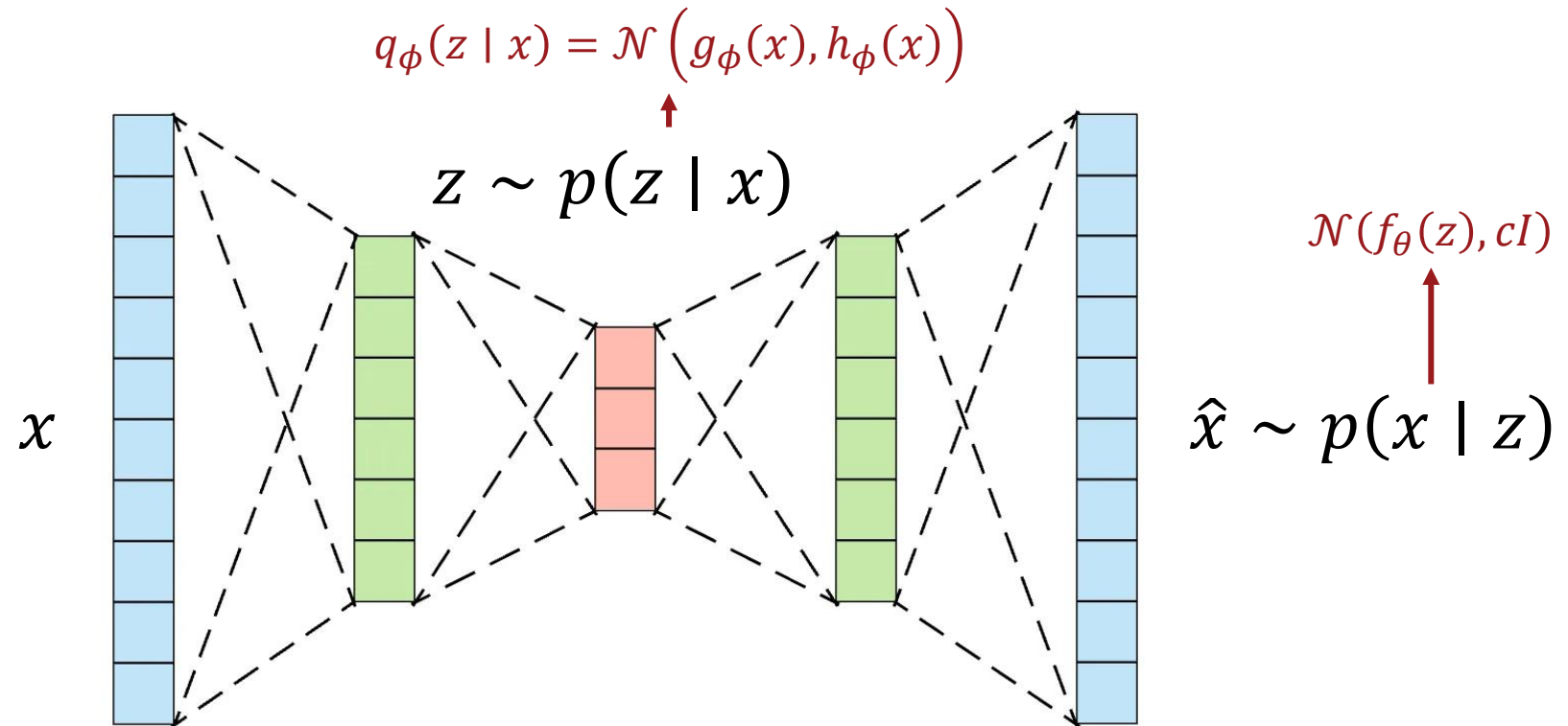
Same as autoencoders, but everything is a distribution now.



Variational autoencoders (VAE)

Just minimize $(x - \hat{x})^2$?

No.



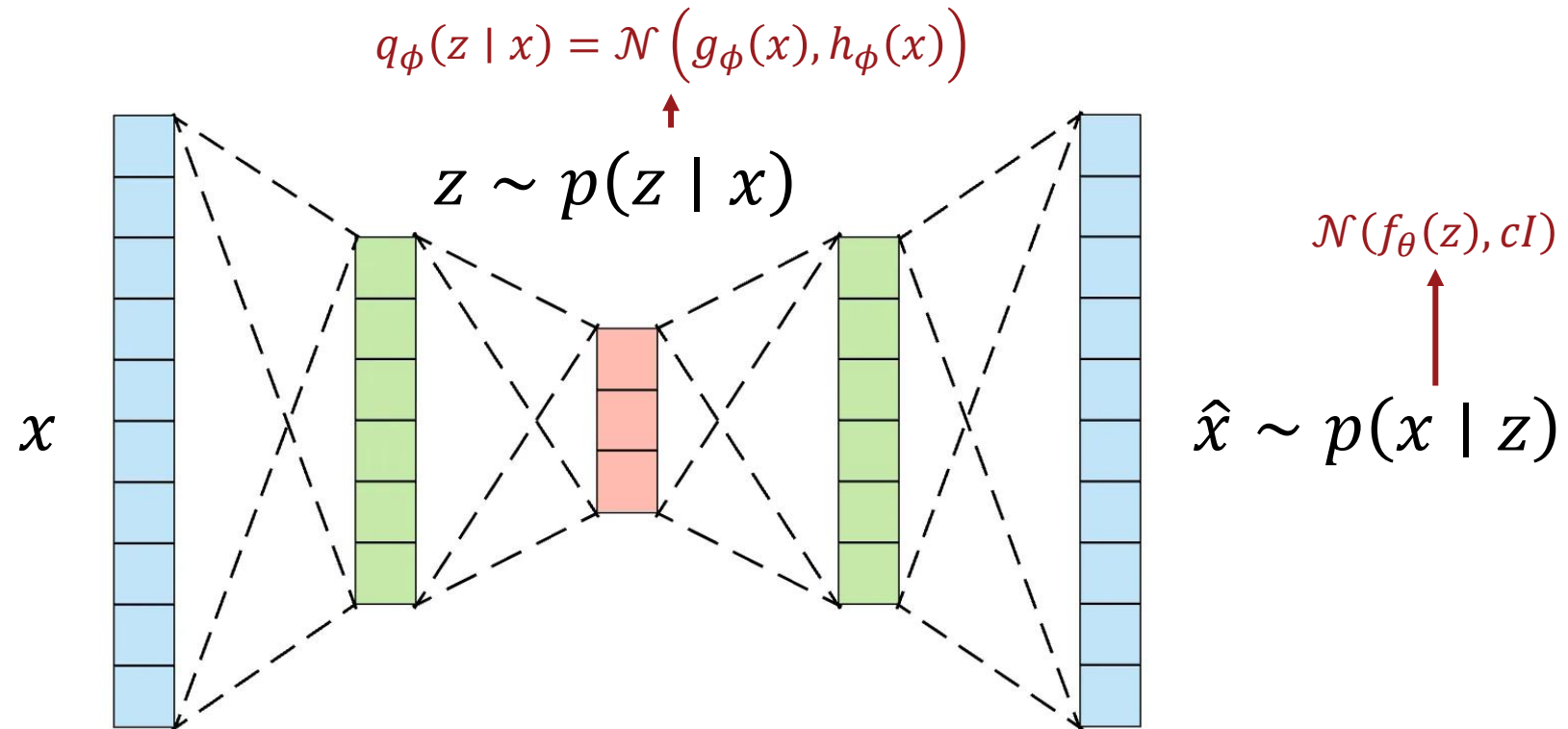
Variational autoencoders (VAE)

Just minimize $(x - \hat{x})^2$?

No.

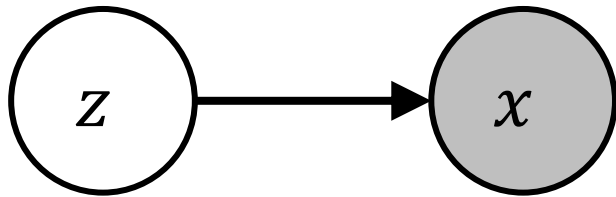
The network would learn Gaussians for z that are far from each other.

We need to regularize.



Variational autoencoders (VAE)

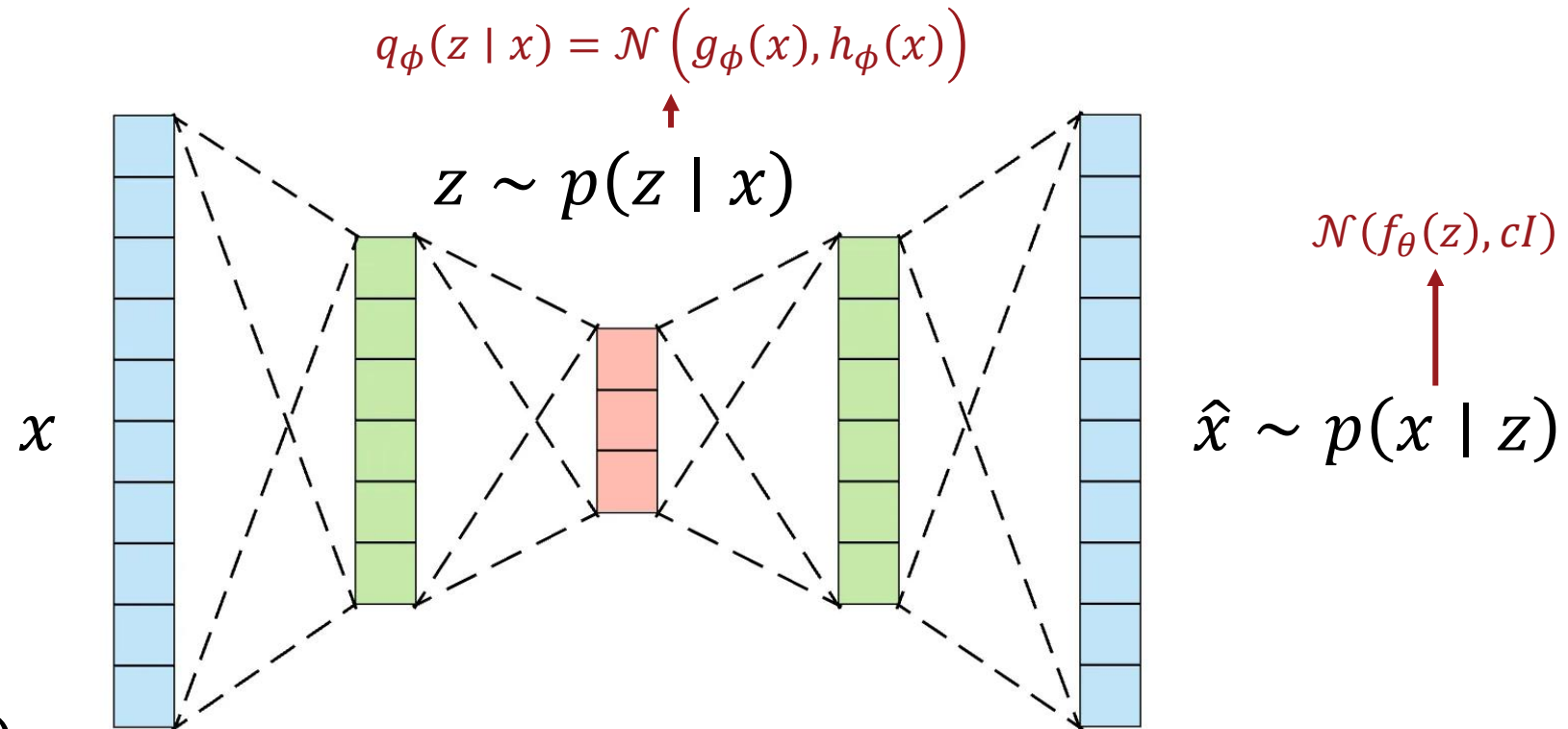
Graphical model



x is observed.

We want to maximize the likelihood of the observed data: $p(x)$.

But we can't compute $p(x)$.



Variational autoencoders (VAE)

$$p(x)$$

Distribution of raw data.

MNIST example:

All samples from $p(x)$ is a drawing of a 0-9 digit. For all such drawings $p(x) > 0$.

We don't have this.

Variational autoencoders (VAE)

$$p(x)$$

Distribution of raw data.

MNIST example:

All samples from $p(x)$ is a drawing of a 0-9 digit. For all such drawings $p(x) > 0$.

We don't have this.

$$p(z)$$

Distribution of the latent code.

We can assume anything.

This is what makes VAEs work!

We will assume $\mathcal{N}(0, I)$ so that the network will not cheat by learning Gaussians that are far.

Variational autoencoders (VAE)

Our goal is to maximize $p(x)$, or equivalently $\log p(x)$.

We will use **variational inference**.

Variational inference

$$D_{KL}(q_\phi(z|x) \parallel p(z|x)) = -\int q_\phi(z|x) \log \left(\frac{p(z|x)}{q_\phi(z|x)} \right) dz \geq 0$$

$$-\int q_\phi(z|x) \log \left(\frac{p_\theta(x|z)p(z)}{q_\phi(z|x)p(x)} \right) dz \geq 0$$

$$-\int q_\phi(z|x) \left[\log \left(\frac{p_\theta(x|z)p(z)}{q_\phi(z|x)} \right) - \log p(x) \right] dz \geq 0$$

$$\log p(x) \int q_\phi(z|x) dz - \int q_\phi(z|x) \log \left(\frac{p_\theta(x|z)p(z)}{q_\phi(z|x)} \right) dz \geq 0$$

Variational inference

$$\log p(x) \geq \int q_{\phi}(z | x) \log \left(\frac{p_{\theta}(x | z) p(z)}{q_{\phi}(z | x)} \right) dz$$

$$\log p(x) \geq \int q_{\phi}(z | x) \left[\log p_{\theta}(x | z) + \log \left(\frac{p(z)}{q_{\phi}(z | x)} \right) \right] dz$$

$$\log p(x) \geq \mathbb{E}_{z \sim q_{\phi}(\cdot | x)} [\log p_{\theta}(x | z)] + \int q_{\phi}(z | x) \log \left(\frac{p(z)}{q_{\phi}(z | x)} \right) dz$$

$$\log p(x) \geq \mathbb{E}_{z \sim q_{\phi}(\cdot | x)} [\log p_{\theta}(x | z)] - D_{KL}(q_{\phi}(z | x) \parallel p(z))$$

Variational inference

$$\log p(x) \geq \underbrace{\mathbb{E}_{z \sim q_\phi(\cdot|x)} [\log p_\theta(x|z)] - D_{KL}(q_\phi(z|x) \parallel p(z))}_{\text{Evidence Lower Bound (ELBO)}}$$

Evidence Lower Bound (ELBO)

Variational inference

$$\log p(x) \geq \underbrace{\mathbb{E}_{z \sim q_\phi(\cdot|x)} [\log p_\theta(x|z)] - D_{KL}(q_\phi(z|x) \parallel p(z))}_{\text{Evidence Lower Bound (ELBO)}}$$

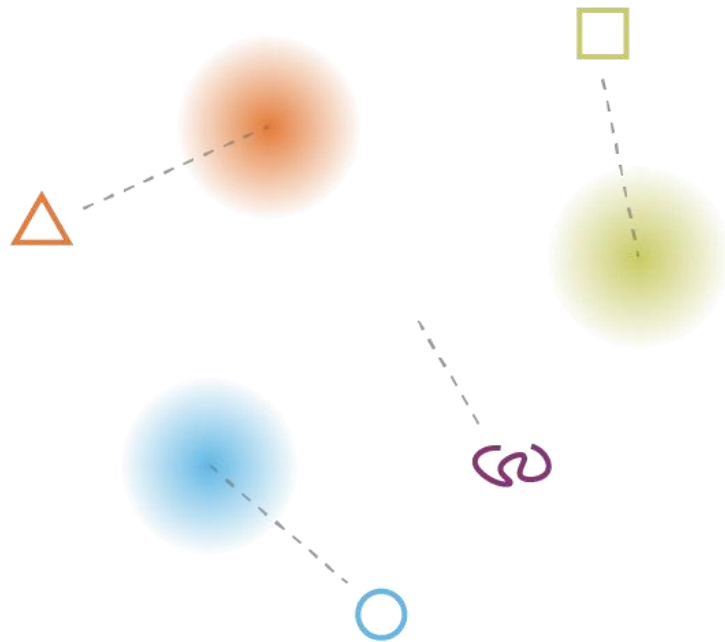
Evidence Lower Bound (ELBO)

Practical version:

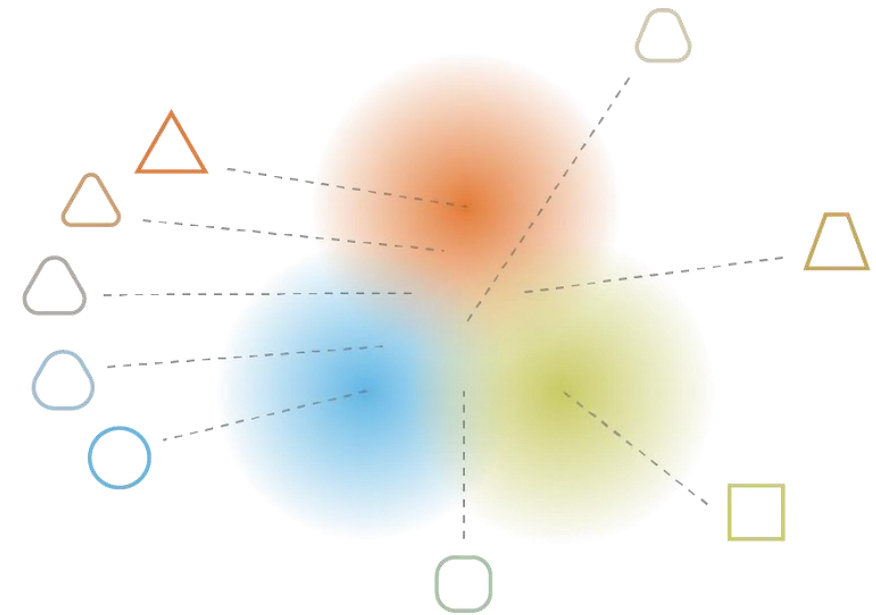
$$\log p(x) \geq \mathbb{E}_{z \sim q_\phi(\cdot|x)} [\log p_\theta(x|z) + \log p(z) - \log q_\phi(z|x)]$$

Comparison

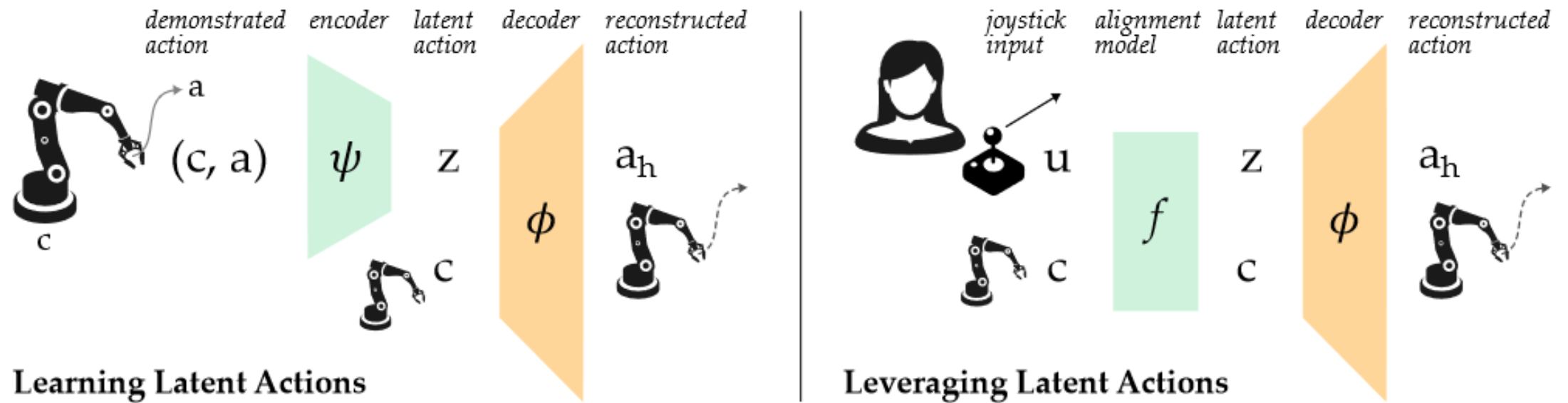
Autoencoder, or
VAE without regularization



VAE with regularization

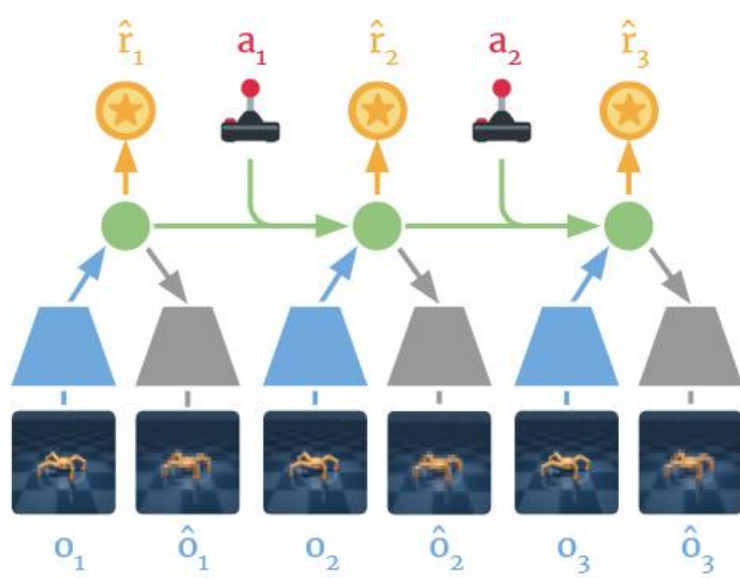


Variational autoencoders in robotics

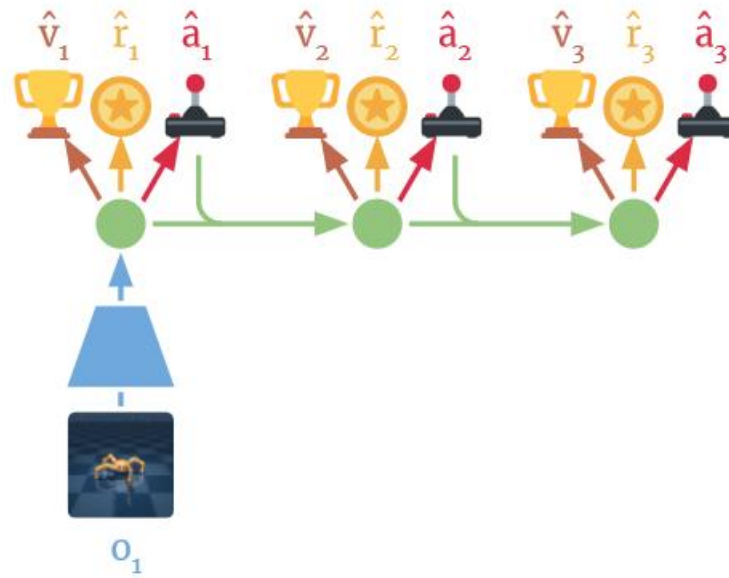


More on this in a few weeks!

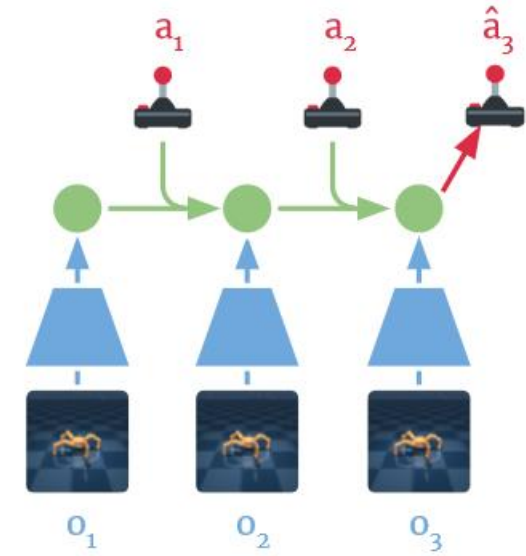
Variational autoencoders in robotics



(a) Learn dynamics from experience



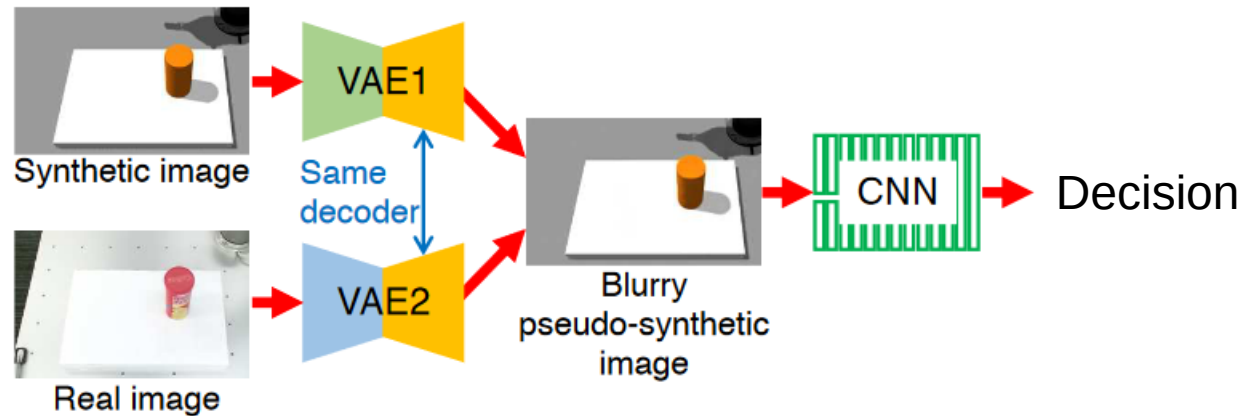
(b) Learn behavior in imagination



(c) Act in the environment

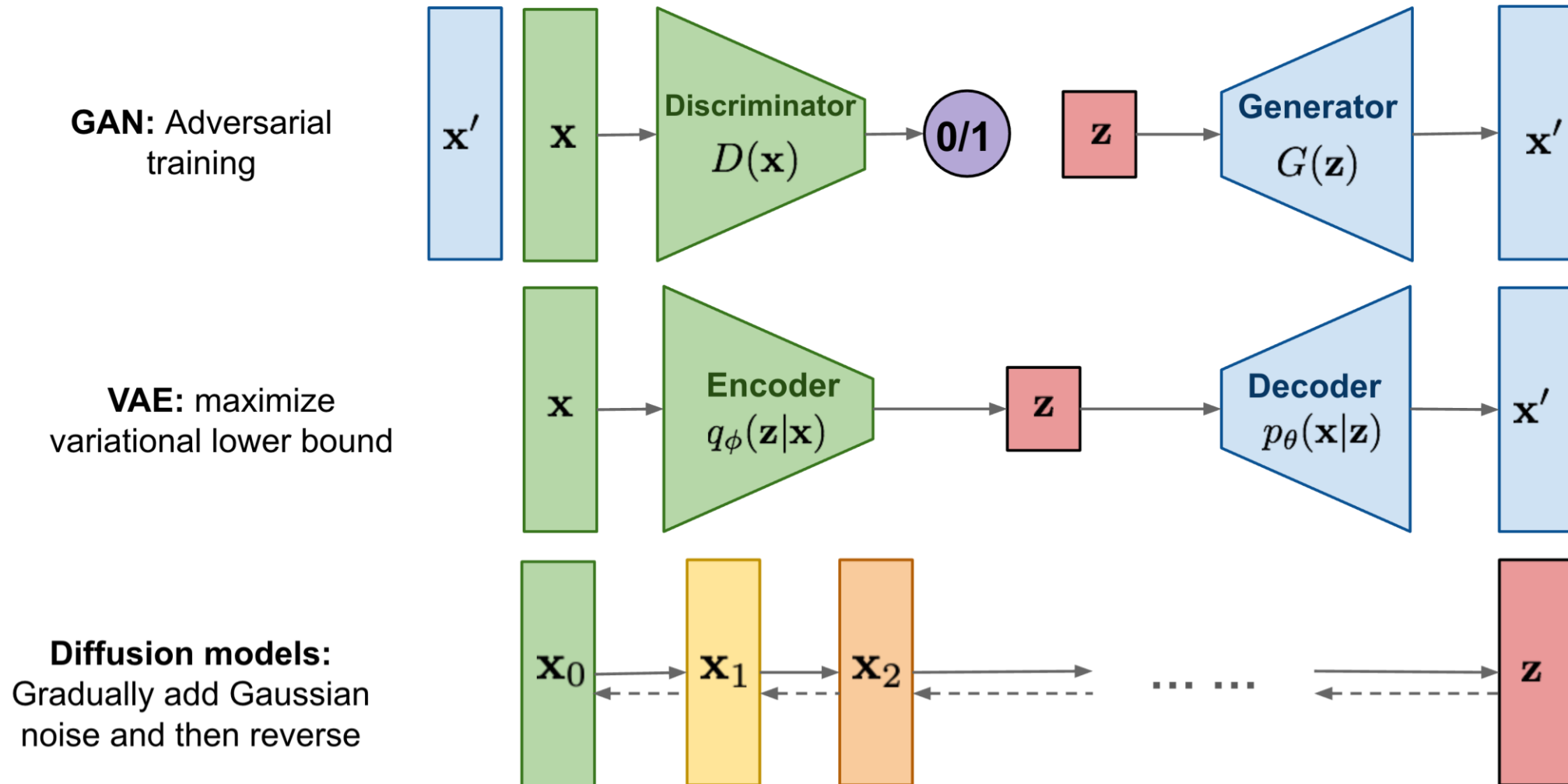
More on this in a few weeks!

Variational autoencoders in robotics

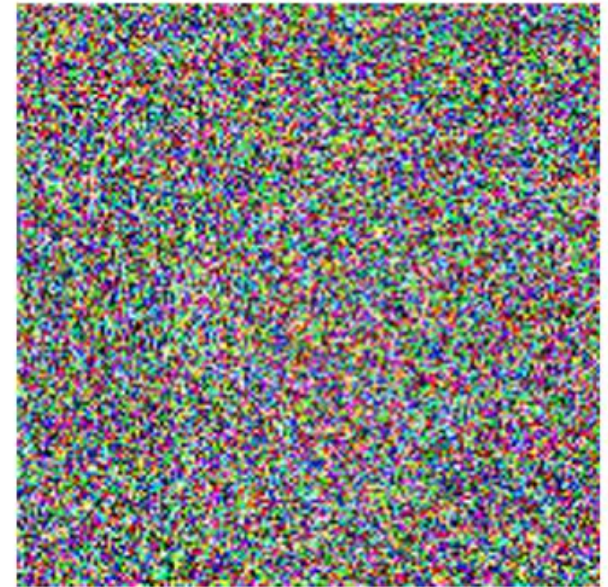
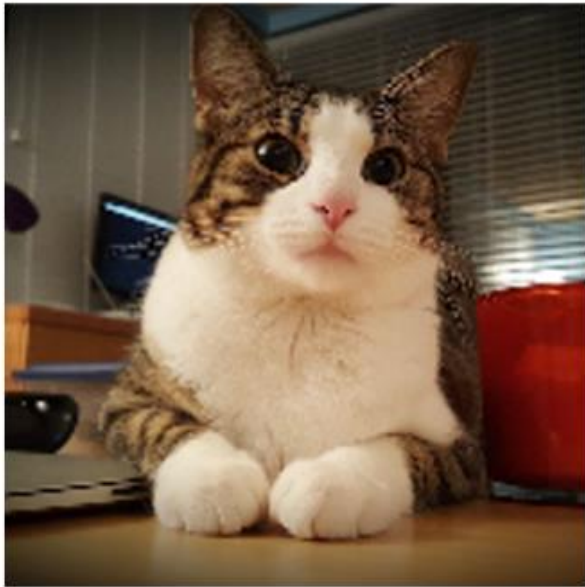


More on this in a few weeks!

Diffusion models



Diffusion models



Diffusion models

$$q(x_{1:T} | x_0) = \prod_{t=1}^T q(x_t | x_{t-1})$$
$$q(x_t | x_{t-1}) = \mathcal{N}\left(x_t; \sqrt{1 - \beta_t} x_{t-1}, \beta_t I\right)$$

Let $\alpha_t = 1 - \beta_t$ and $\bar{\alpha}_t = \prod_{i=1}^t \alpha_i$

$$x_t = \sqrt{\alpha_t} x_{t-1} + \sqrt{1 - \alpha_t} \epsilon \text{ where } \epsilon \sim \mathcal{N}(0, I)$$

$$x_t = \sqrt{\alpha_t \alpha_{t-1}} x_{t-2} + \sqrt{1 - \alpha_t \alpha_{t-1}} \epsilon$$

$$x_t = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$$

$$q(x_t | x_0) = \mathcal{N}\left(x_t; \sqrt{\bar{\alpha}_t} x_0, (1 - \bar{\alpha}_t) I\right)$$

Diffusion models

We will try to learn a reverse diffusion process:

$$p_{\theta}(x_{t-1} \mid x_t) = \mathcal{N}(x_{t-1}; \mu_{\theta}(x_t, t), \Sigma_{\theta}(x_t, t))$$

How do we do this?

Think of it analogous to VAEs.

x was our data sample, and z was its latent representation.

x_0 is our data sample, and x_T is its latent representation.

Diffusion models

So... Maximize the probability of data $p_\theta(x_0)$ with similar tricks:

$$\log p_\theta(x_0) \geq \log p_\theta(x_0) - D_{KL}(q(x_{1:T} | x_0) \| p_\theta(x_{1:T} | x_0))$$

$$= \log p_\theta(x_0) - \mathbb{E}_q \left[\log \frac{q(x_{1:T} | x_0)}{p_\theta(x_{0:T})/p_\theta(x_0)} \right]$$

$$= \log p_\theta(x_0) - \mathbb{E}_q \left[\log \frac{q(x_{1:T} | x_0)}{p_\theta(x_{0:T})} + \log p_\theta(x_0) \right]$$

$$= -\mathbb{E}_q \left[\log \frac{q(x_{1:T} | x_0)}{p_\theta(x_{0:T})} \right]$$

Diffusion models

$$\begin{aligned}\log p_{\theta}(x_0) &\geq -\mathbb{E}_q \left[\log \frac{q(x_{1:T} \mid x_0)}{p_{\theta}(x_{0:T})} \right] \\&= -\mathbb{E}_q \left[\log \frac{\prod_{t=1}^T q(x_t \mid x_{t-1})}{p_{\theta}(x_T) \prod_{t=1}^T p_{\theta}(x_{t-1} \mid x_t)} \right] \\&= -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \sum_{t=1}^T \log \frac{q(x_t \mid x_{t-1})}{p_{\theta}(x_{t-1} \mid x_t)} \right] \\&= -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \sum_{t=2}^T \log \frac{q(x_t \mid x_{t-1})}{p_{\theta}(x_{t-1} \mid x_t)} + \log \frac{q(x_1 \mid x_0)}{p_{\theta}(x_0 \mid x_1)} \right]\end{aligned}$$

Diffusion models

$$\begin{aligned}\log p_{\theta}(x_0) &\geq -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \sum_{t=2}^T \log \frac{q(x_t | x_{t-1})}{p_{\theta}(x_{t-1} | x_t)} + \log \frac{q(x_1 | x_0)}{p_{\theta}(x_0 | x_1)} \right] \\ &= -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \underbrace{\sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0) q(x_t | x_0)}{p_{\theta}(x_{t-1} | x_t) q(x_{t-1} | x_0)}}_{\sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0)}{p_{\theta}(x_{t-1} | x_t)}} + \log \frac{q(x_1 | x_0)}{p_{\theta}(x_0 | x_1)} \right] \\ &\quad \sum_{t=2}^T \log \frac{q(x_t | x_0)}{q(x_{t-1} | x_0)}\end{aligned}$$

Diffusion models

$$\begin{aligned}\log p_{\theta}(x_0) &\geq -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \sum_{t=2}^T \log \frac{q(x_t | x_{t-1})}{p_{\theta}(x_{t-1} | x_t)} + \log \frac{q(x_1 | x_0)}{p_{\theta}(x_0 | x_1)} \right] \\ &= -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \underbrace{\sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0) q(x_t | x_0)}{p_{\theta}(x_{t-1} | x_t) q(x_{t-1} | x_0)}}_{\sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0)}{p_{\theta}(x_{t-1} | x_t)}} + \log \frac{q(x_1 | x_0)}{p_{\theta}(x_0 | x_1)} \right] \\ &\quad \sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0)}{p_{\theta}(x_{t-1} | x_t)} + \log \frac{q(x_T | x_0)}{q(x_1 | x_0)}\end{aligned}$$

See the cancellation

Diffusion models

$$\log p_{\theta}(x_0) \geq -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0)}{p_{\theta}(x_{t-1} | x_t)} + \log \frac{q(x_T | x_0)}{p_{\theta}(x_0 | x_1)} \right]$$

Remember this? It's just a Gaussian!

$$= -\mathbb{E}_q \left[\log \frac{q(x_T | x_0)}{p_{\theta}(x_T)} + \sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0)}{p_{\theta}(x_{t-1} | x_t)} - \log p_{\theta}(x_0 | x_1) \right]$$

And this? It's the standard normal!

How about this?

These are just the reverse process networks we are learning!

$$q(x_{t-1} | x_t, x_0) = q(x_t | x_{t-1}, x_0) \frac{q(x_{t-1} | x_0)}{q(x_t | x_0)}$$

Diffusion models

$$\log p_{\theta}(x_0) \geq -\mathbb{E}_q \left[-\log p_{\theta}(x_T) + \sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0)}{p_{\theta}(x_{t-1} | x_t)} + \log \frac{q(x_T | x_0)}{p_{\theta}(x_0 | x_1)} \right]$$

Remember this? It's just a Gaussian!

$$= -\mathbb{E}_q \left[\log \frac{q(x_T | x_0)}{p_{\theta}(x_T)} + \sum_{t=2}^T \log \frac{q(x_{t-1} | x_t, x_0)}{p_{\theta}(x_{t-1} | x_t)} - \log p_{\theta}(x_0 | x_1) \right]$$

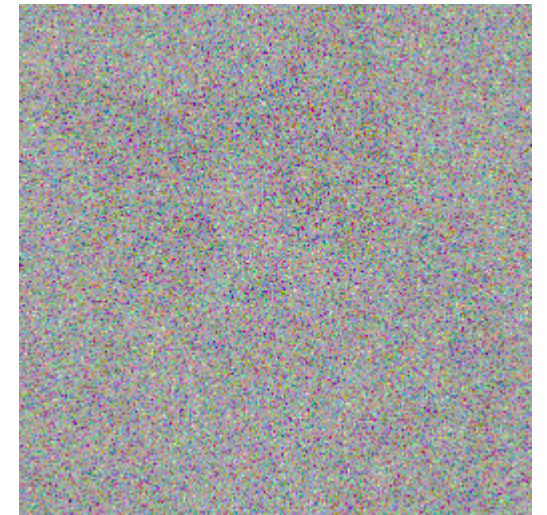
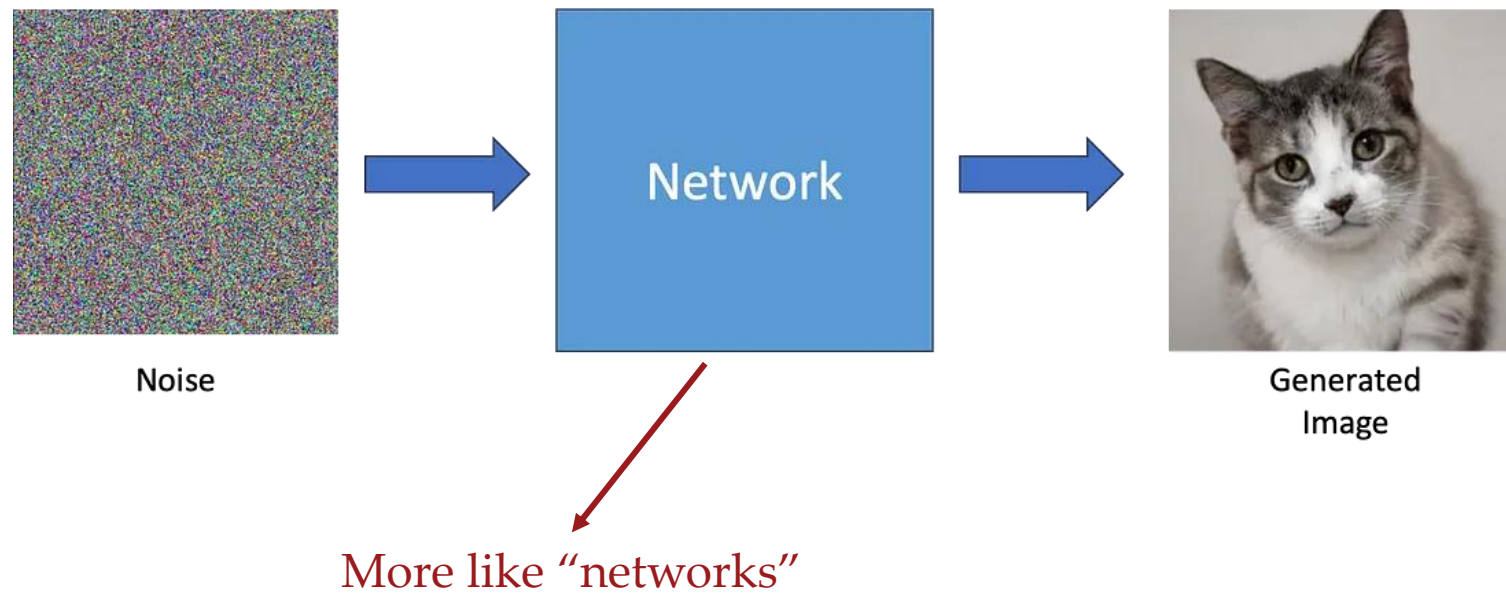
And this? It's the standard normal!

How about this?

These are just the reverse process networks we are learning!

$$q(x_{t-1} | x_t, x_0) = q(x_t | x_{t-1}) \frac{q(x_{t-1} | x_0)}{q(x_t | x_0)}$$

Diffusion models



Today

- Basics of computer vision for robotics
- Representation learning

Next time...

- Presentations on representation learning
- Reinforcement learning